# P22 HW 5

① a-c are of the form $\frac{a}{x+iy} \cdot \frac{x-iy}{x-iy} = \left(\frac{ax}{x^2+y^2}\right) - i\left(\frac{ay}{x^2+y^2}\right)$

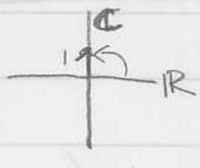
a) $\boxed{\frac{29}{20+21i}} = \left(\frac{29 \cdot 20}{20^2+21^2}\right) - i\left(\frac{29 \cdot 21}{20^2+21^2}\right) = \left(\frac{20}{29}\right) - i\left(\frac{21}{29}\right)$

b) $\frac{1}{1+i} = \frac{1}{2} - \frac{i}{2} = \frac{1}{2}(1-i)$

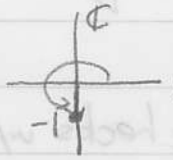
c) $\frac{5}{-3+5i} = \left(\frac{5\cdot(-3)}{9+25}\right) - \left(\frac{5.5}{9+25}\right)i = \left(\frac{-15}{36}\right) - \left(\frac{25}{36}\right)i$

d) $\sqrt{i} = \left(e^{\pi i/2}\right)^{1/2} = e^{\pi i/4} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(1+i)$

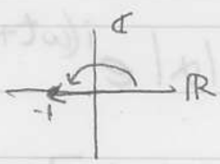
②

a) 

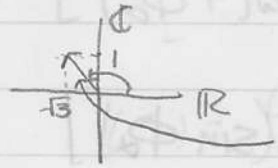

$r=1,\ \theta = \frac{\pi}{2} \Rightarrow z = e^{\pi i/2}$

b) 


$r=1,\ \theta = \frac{3\pi}{2} \Rightarrow z = e^{3\pi i/2}$

c) 


$r=1,\ \theta = \pi \Rightarrow z = e^{\pi i}$

d) 


$r = \sqrt{1+(\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\left[\frac{y}{x}\right] = \tan^{-1}\left[\frac{1}{\sqrt{3}}\right] = 30° = \frac{\pi}{6}$

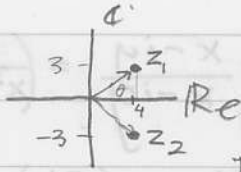
but the $\theta$ we need is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

so $\boxed{z = 2e^{5\pi i/6}}$

③ $z^2 - 8z + 25 = 0 \xrightarrow{\text{quadratic formula}} z = \frac{1}{2}\left[8 \pm \sqrt{64-100}\right] = \frac{8 \pm 6i}{2}$

Imaginary solutions come in pairs $z = 4 \pm 3i$ : $z_1 = 4+3i$, $z_2 = 4-3i$

(3) cont)



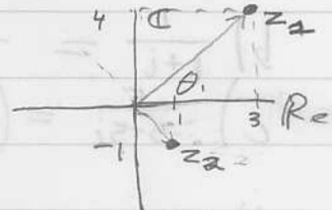

$$\boxed{z_1 = 4+3i, \quad z_2 = 4-3i} \Rightarrow r=5$$

$$\boxed{\theta = \tan^{-1}\left[\frac{y}{x}\right] = \tan^{-1}\left(\frac{3}{4}\right) = 0.6435}$$

$$\boxed{z_1 = 5e^{i\theta} \qquad z_2 = 5e^{-i\theta}}$$

(4) $z_1 = 3+4i$ $\qquad r_1 = \sqrt{3^2+4^2} = 5$

$z_2 = 1-i$ $\qquad r_2 = \sqrt{1^2+1^2} = \sqrt{2}$




$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 0.927$ $\qquad z_1 = r_1 e^{i\theta_1}$

$\theta_2 = \tan^{-1}\left(\frac{y_2}{x_2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4} = -0.785$ $\qquad z_2 = r_2 e^{i\theta_2}$

$z_3 \equiv z_1 z_2 = (3+4i)(1-i) = 3-3i+4i+4 = 7+i$

$r_3 = \sqrt{7^2+1} = 5\sqrt{2} = r_1 r_2$

$\theta_3 = \tan^{-1}\left[\frac{1}{7}\right] = 0.141$ $\qquad \theta_1+\theta_2 = 0.141$ ✓

So $z_3 = z_1 z_2 = (r_1 r_2) e^{i(\theta_1+\theta_2)} = (5\sqrt{2})\, e^{i\theta_3}$ $\quad$ checks w/ Cartesia

(5) $x(t) = -5\sin(\omega t) + 12\cos(\omega t) = \mathrm{Re}\left[|\alpha| e^{i(\omega t+\phi_\alpha)}\right]$

$|\alpha|\,\mathrm{Re}\left[e^{i(\omega t+\phi_\alpha)}\right] = |\alpha|\,\underline{\mathrm{Re}\left[\cos(\omega t+\phi_\alpha)\right.} + i\sin(\omega t+\phi_\alpha)\left.\right]$

$|\alpha|\cos(\omega t+\phi_\alpha) = |\alpha|\left[\cos(\omega t)\cos(\phi_\alpha) - \sin(\omega t)\sin(\phi_\alpha)\right]$

So $\quad -|\alpha|\sin(\phi_\alpha) = -5$

$\quad |\alpha|\cos(\phi_\alpha) = 12$ $\quad \Longrightarrow \quad \boxed{\begin{array}{l}\tan(\phi_\alpha) = \frac{5}{12} \\ \phi_\alpha = \tan^{-1}\left[\frac{-5}{12}\right] = 0.395\end{array}}$

and $\quad \boxed{\begin{array}{l}|\alpha| = \frac{5}{\sin(\phi_\alpha)} = 13 \\ |\alpha| = \frac{12}{\cos(\phi_\alpha)} = 13\end{array}}$ ✓