

① RHK4Q8

$$y = A(x - vt) \quad y = A(x + vt)^2$$

$$y = A\sqrt{x - vt} \quad y = A \ln(x + vt)$$

These are, in general, unbounded in  $y$ ...  $y$  can go to  $-\infty$  and up to  $+\infty$ ... real waves generally cannot have amplitudes so large. In addition  $\sqrt{x - vt}$  and  $\ln(x + vt)$  are complex-valued for  $x - vt < 0$  or  $x + vt < 0$ , which is peculiar... sometimes the real part of  $y$  is interpreted as the actual amplitude, though.

② RHK4P2 (a)  $\nu = \frac{12}{30} = \frac{2}{5} = 0.4 \text{ Hz}$

(b)  $v = \frac{15 \text{ m}}{5 \text{ s}} = 3 \text{ m/s}$

(c)  $\lambda \nu = v$

$$\lambda = \frac{v}{\nu} = \frac{3 \text{ m/s}}{0.4 \text{ 1/s}} = 7.5 \text{ m}$$

③ RHK4P6  $\cos(kx - \omega t)$

$$v\lambda = v$$

$$\nu = 493 \text{ Hz} \quad \lambda = \frac{v}{\nu} = \frac{353 \text{ m/s}}{493 \text{ 1/s}} = 0.716 \text{ m}$$

$$v = 353 \text{ m/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.716} = 8.78 \text{ 1/m}$$

(a)  $t = \text{same}$ 

$$\text{phase} = 55^\circ = \frac{\pi}{180} \cdot 55 = 0.96 \text{ rad}$$

$$k\Delta x = 0.96 \text{ rad}$$

$$\Delta x = \frac{0.96 \text{ rad}}{8.78 \text{ /m}} = 0.109 \text{ m}$$

$$(b) \omega = 2\pi\nu = 2\pi \cdot 493 \text{ /s}$$

$$\omega = 3098 \text{ rad/s}$$

$$\text{phase} = \omega\Delta t = 3098 \cdot 1.12 \cdot 10^{-3}$$

$$= 3.47 \text{ radians}$$

$$\text{phase} = \frac{180}{\pi} \cdot 3.47 = 199^\circ$$

Extra RHK 4 P7

$$(a) y(x,t) = y_m \sin(kx - \omega t - \phi)$$

$$\frac{\partial y}{\partial t} = y_m \cos(kx - \omega t - \phi) (-\omega)$$

maximum  $\cos = 1 \dots$ 

$$\left(\frac{\partial y}{\partial t}\right)_{\text{max}} = \omega y_m$$

Maximum

$$(b) \frac{\partial^2 y}{\partial t^2} = -\omega y_m (-\sin(kx - \omega t - \phi) (-\omega)) \rightarrow = \omega^2 y_m$$

4) RHYA P9

(a)  $y = 6.0 \sin(0.020\pi x + 4\pi t)$

$y_m = 6.0 \text{ cm}$

$k = 0.020\pi$        $\omega = 4\pi$

amplitude =  $y_m = 6.0 \text{ cm}$

(b)  $k = \frac{2\pi}{\lambda} = 0.020\pi$

$\lambda = \frac{2}{0.02} = 100 \text{ cm}$

(c)  $\nu = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2 \text{ } 1/\text{s} = 2 \text{ Hz}$

(d)  $\lambda \nu = \text{speed} = 200 \text{ cm/s}$

(e) + sign  $\rightarrow$  right to left

or  $0.020\pi x + 4.0\pi t = \text{constant}$

$0.020\pi v + 4.0\pi = 0$

$v = \frac{-4.0}{0.02} = -200 \frac{\text{cm}}{\text{s}}$

- velocity is right to left

(f) problem 7  $\rightarrow v_{\text{max}} = \omega y_m$

$v_{\text{max}} = 4\pi \cdot 6 = 24\pi \text{ cm/s}$   
 $= 75.4 \text{ cm/s}$

5) RHK4 P10

$$\mu = \frac{0.625 \text{ kg}}{2.15 \text{ m}} = 0.291 \text{ kg/m}$$

$$F = 487 \text{ N} = 487 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{487 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{0.291 \text{ kg/m}}} = 40.9 \frac{\text{m}}{\text{s}}$$

6) RHK4 P17

$$v_{\text{max}} = \frac{\partial y}{\partial t} = y_m (\underbrace{\sin(kx - \omega t)}_{\text{max} = 1}) (\omega)$$

$$\frac{v_{\text{max}}}{v} = \frac{\omega y_m}{v}$$

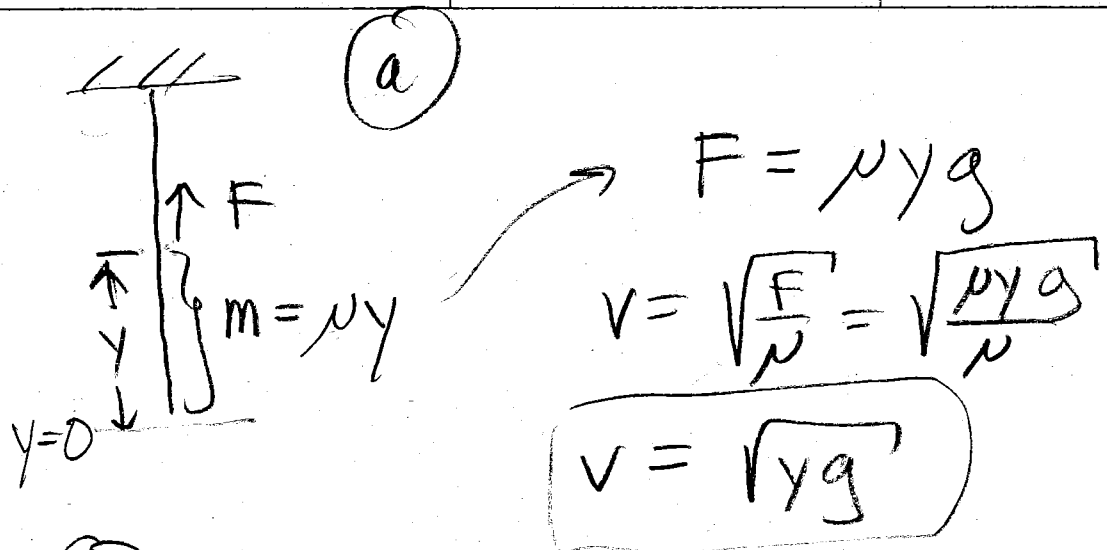
$$\frac{v_{\text{max}}}{v} = \omega y_m \sqrt{\frac{\mu}{F}}$$

Independent variables, can be chosen independent of  $\mu, F$

does depend on what the cord is made of

7) RHK4 P22

Key point: tension  $F$  depends on  $y$



(b)  $dy = v dt$        $v$  is a function of  $y$ !

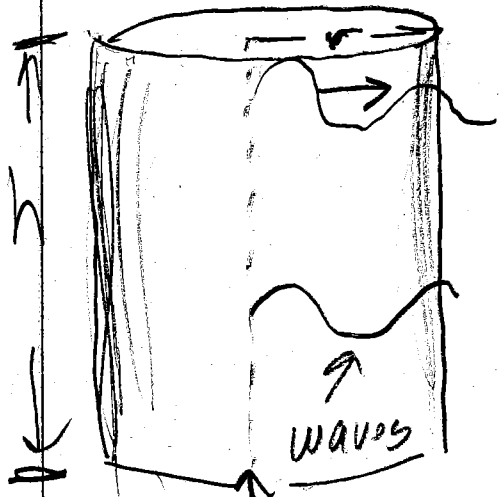
$$\frac{dy}{\sqrt{y g}} = dt'$$

$$\int_0^L \frac{dy}{\sqrt{y g}} = \int_0^t dt' = t$$

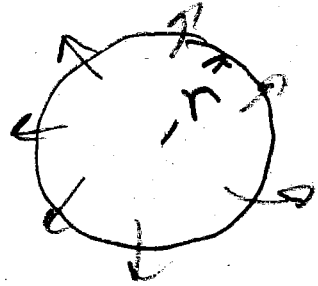
$$\frac{1}{\sqrt{g}} 2\sqrt{y} \Big|_0^L = 2\sqrt{\frac{L}{g}} = t$$

(c) NO!

8) RHY4 P26



← waves, look from above!



← Power through circle is constant

Source Emits Power

$$P \propto h$$

$$I \cdot \text{Area} = \text{Power}$$

$$I \cdot 2\pi r \cdot h = \text{Power}$$

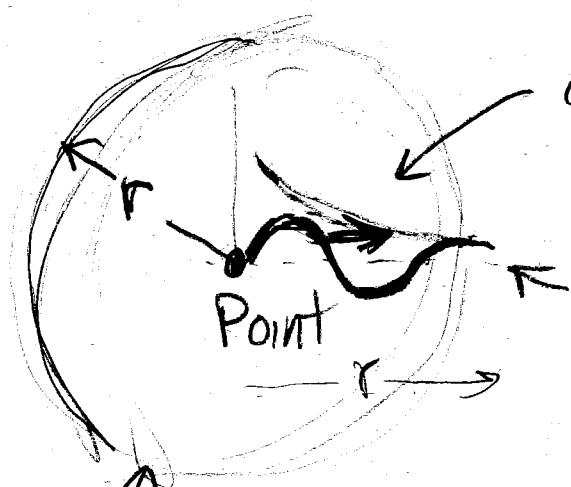
(a)  $I \propto \frac{1}{r}$

(b)  $I \propto y_m^2 \propto \frac{1}{r}$

$$y_m \propto \frac{1}{\sqrt{r}}$$

8) RHK4 P27

(a) Idea is that Power passing through a pertinent boundary is constant. in 3-dimensions the boundary is a spherical surface.



amplitude of wave  
Power  
wave

(constant)<sub>1</sub> y<sub>m</sub><sup>2</sup> (Area)

Surface Area 4πr<sup>2</sup>

= (another constant)<sub>2</sub>

y<sub>m</sub> ∝ √(1 / 4πr<sup>2</sup>)

y<sub>m</sub> ∝ 1/r

More generally,

Power ∝ v (speed)

∝ ω<sup>2</sup>

∝ ρ (mass/volume)

Dimensionally, then, check.

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{\text{mass} \cdot \text{l}^2}{\text{time}^3} \stackrel{?}{=} \underset{\substack{\uparrow \\ \text{l}}}{\text{v}} \cdot \underset{\substack{\uparrow \\ \text{time}}}{\omega^2} \cdot \underset{\substack{\uparrow \\ \text{m}}}{\rho} \cdot \underbrace{\text{y}_m^2 \cdot \text{A}}_{\text{l}^4}$$

$$\frac{\text{m} \cdot \text{l}^2}{\text{t}^3} \stackrel{?}{=} \frac{\text{m} \cdot \text{l}^2}{\text{t}^3} \quad \checkmark$$

so, (constant),  $\propto v \omega^2 \rho$

and  $y_m = \frac{Y}{r}$ , but

(b) dimensions of  $Y$  must be  $(\text{length})^2$



9 PTK4 P 33

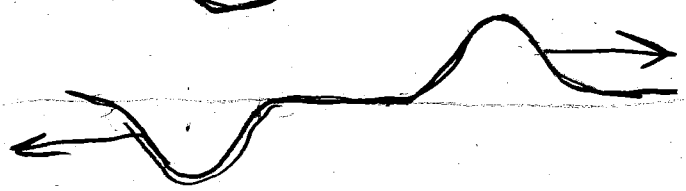
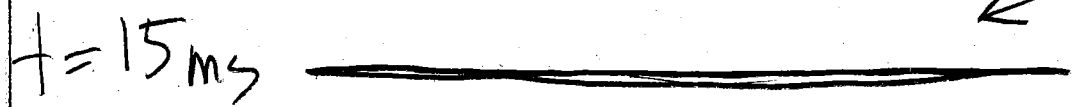
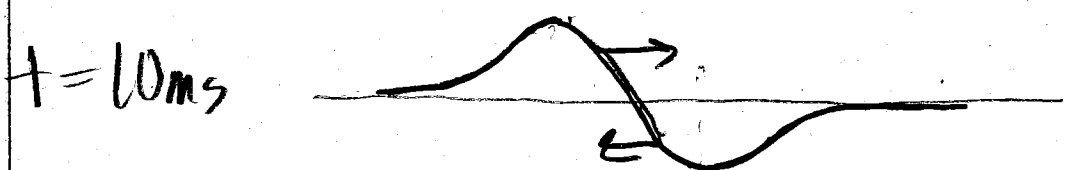
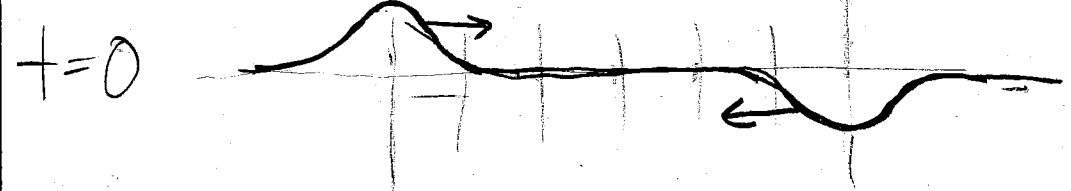
$$v = 2.0 \text{ m/s} = 200 \text{ cm/s}$$

$$= 0.2 \text{ cm/ms}$$

a

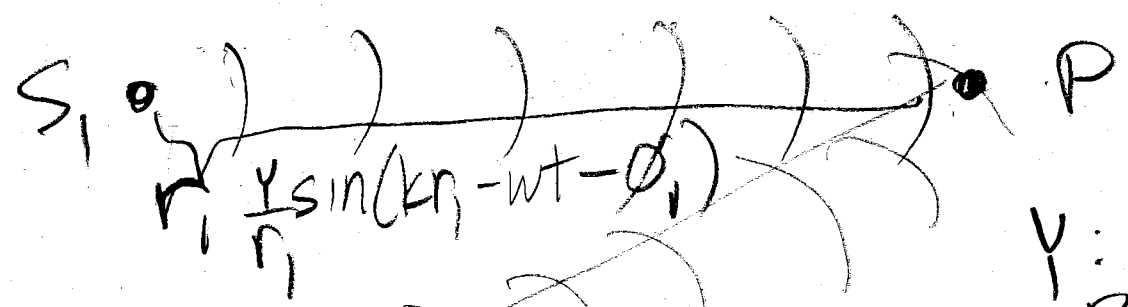
t (ms)	$\Delta x = vt$
5	1.0 cm
10	2.0 cm
15	3.0
20	4.0
25	5.0

1cm → | ←

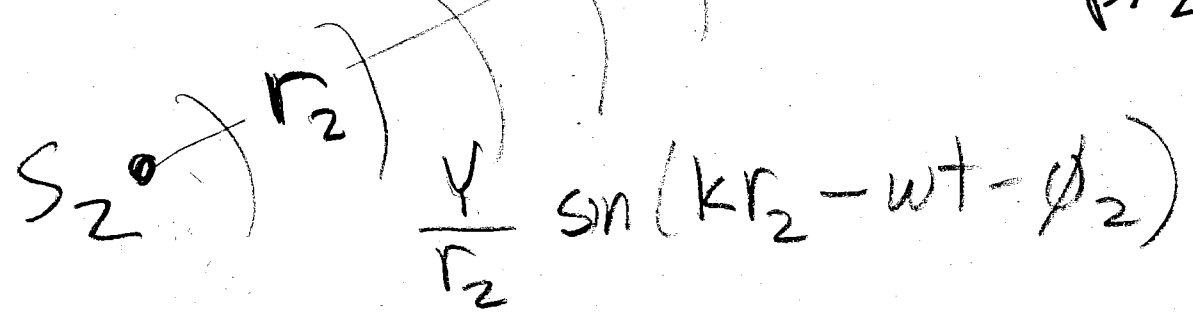


b  
energy  
all  
kinetic

10) RHK4 P36



y: see p. 27



(a) Superposition

$$y(P) = y \left( \frac{1}{r_1} \sin(kr_1 - \omega t - \phi_1) + \frac{1}{r_2} \sin(kr_2 - \omega t - \phi_2) \right)$$

(i)  $\phi_1 = \phi_2$  "same phase"

(ii) set  $t=0$  so  $\phi_1 = \phi_2 = 0$

$$r \equiv \frac{1}{2}(r_1 + r_2) \quad \Delta r \equiv r_1 - r_2$$

$$\text{then } r_1 = r + \frac{1}{2} \Delta r$$

$$r_2 = r - \frac{1}{2} \Delta r$$

$$\frac{1}{r_1} = \frac{1}{r + \frac{1}{2} \Delta r} \approx \frac{1}{r} \left( 1 - \frac{1}{2} \frac{\Delta r}{r} \right)$$

$$\frac{1}{r_2} = \frac{1}{r - \frac{1}{2}\Delta r} \approx \frac{1}{r} \left(1 + \frac{1}{2} \frac{\Delta r}{r}\right)$$

so,

$$y(P) \approx \frac{Y}{r} \left[ \sin(kr_1 - \omega t) + \sin(kr_2 - \omega t) \right. \\ \left. - \frac{\Delta r}{2r} \left( \sin(kr_1 - \omega t) - \sin(kr_2 - \omega t) \right) \right]$$

eq. 35

$$\sin B + \sin C = 2 \sin \frac{1}{2}(B+C) \cos \frac{1}{2}(B-C)$$

$$\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)$$

$$= 2 \sin \left[ \frac{k}{2}(r_1 + r_2) - \omega t \right] \left[ \cos \frac{k}{2}(r_1 - r_2) \right]$$

$$\sin(kr_1 - \omega t) - \sin(kr_2 - \omega t)$$

$$= 2 \sin \frac{k}{2}(r_1 - r_2) \cos \left[ \frac{k}{2}(r_1 + r_2) - \omega t \right]$$

so,

$$y(P) \approx \frac{2Y}{r} \left[ \cos \frac{k}{2}(r_1 - r_2) \sin \left[ \frac{k}{2}(r_1 + r_2) - \omega t \right] \right.$$

$$\left. - \frac{\Delta r}{2r} \sin \frac{k}{2}(r_1 - r_2) \cos \left[ \frac{k}{2}(r_1 + r_2) - \omega t \right] \right]$$

$$y(P) = \frac{2Y}{r} \left[ \cos \frac{k\Delta r}{2} \sin[kr - \omega t] - \frac{1}{2} \frac{\Delta r}{r} \sin \frac{k\Delta r}{r} \times \cos(kr - \omega t) \right]$$

Idea is:  $\frac{\Delta r}{r}, \sin \frac{k\Delta r}{r}$  is second order in  $\frac{\Delta r}{r}$  and negligible

$$y(P) = \underbrace{\left( \frac{2Y}{r} \cos \frac{k\Delta r}{2} \right)}_{Y_m} \sin(kr - \omega t)$$

(b) Cancellation:  $\frac{k\Delta r}{2} = \underbrace{\left( n + \frac{1}{2} \right) \pi}_{\text{integer}}$

$$\frac{\cancel{2\pi}}{\lambda} \frac{\Delta r}{2} = \left( n + \frac{1}{2} \right) \pi$$

$$\Delta r = r_1 - r_2 = \left( n + \frac{1}{2} \right) \lambda$$

Reinforcement  $\frac{k\Delta r}{2} = n\pi$

so  $\Delta r = r_1 - r_2 = n\lambda$

11) RHK4 P40

$$\mu = 7.16 \text{ g/m} \quad F = 152 \text{ N}$$

$$L = 89.4 \text{ cm}$$

$$(a) \quad v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{152 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{7.16 \cdot 10^{-3} \frac{\text{kg}}{\text{m}}}}$$

$$v = 145 \text{ m/s}$$

$$(b) \quad \lambda = \frac{2}{3} L = \frac{2}{3} \cdot 0.894 \text{ m}$$

$$\lambda = 0.596 \text{ m}$$

$$(c) \quad v \lambda = v \quad \text{so}$$

$$v = \frac{v}{\lambda} = \frac{145 \text{ m/s}}{0.596 \text{ m}} = 244 \text{ Hz}$$

12) RHK4 P48

$$n=1, \text{ so } \lambda = 2L = 2 \cdot 15 = 0.30 \text{ m}$$

$$v = 250 \text{ m/s}$$

$$(a) \quad \frac{\text{string}}{\lambda v} = v, \quad v = \frac{v}{\lambda} = \frac{250 \text{ m/s}}{0.30 \text{ m}}$$
$$v = 833 \text{ Hz}$$

$$(b) \quad \frac{\text{air}}{\lambda_a v} = v_a \Rightarrow \lambda_a = \frac{v_a}{v} = \frac{348}{833} = 0.418 \text{ m}$$