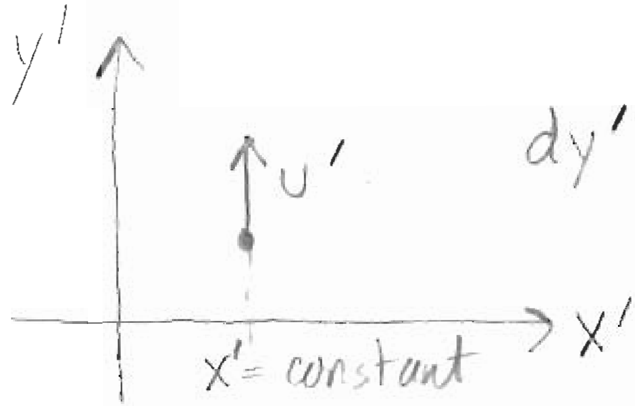
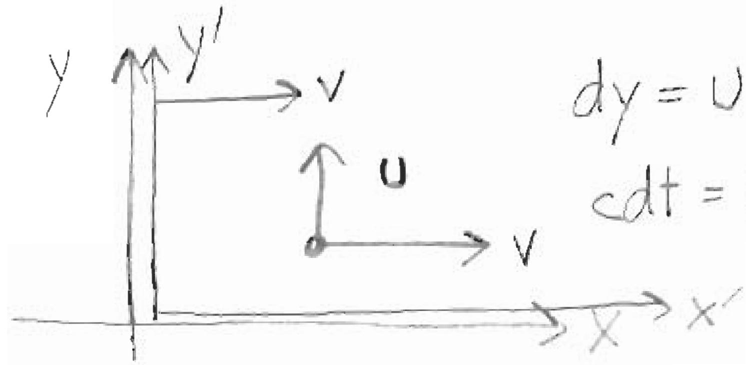
Momentum

First: transformation of $\perp$ velocities



$$dy' = u'\,dt'$$



$$dy = u\,dt = dy' = u'\,dt'$$

$$c\,dt = \gamma(c\,dt' + \beta \underbrace{dx'}_{\substack{0 \\ \text{constant}}})$$

$$dt = \gamma\,dt$$

$$u \cdot \gamma\,dt' = u'\,dt'$$

$$\boxed{u = \frac{u'}{\gamma}}$$ when u' is $\perp$ to boost

or $\beta_u = \dfrac{\beta_u'}{\gamma}$ $\quad\perp$

$\parallel \qquad \beta_u = \dfrac{\beta_u' + \beta_v}{1 + \beta_u'\beta_v} \qquad \parallel$

Non-relatistic

$$p = mv$$

Quandary: when $v \to c$, seems like $p_{max} = mc$. What happens if you keep pushing?

$$\frac{dp}{dt} = F$$ ← need not be zero.

Only way out:

$$p = \underline{m(v)}\, v$$

will turn out to be ...

$$\underline{m(v)} = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma m_0$$

old days the mass

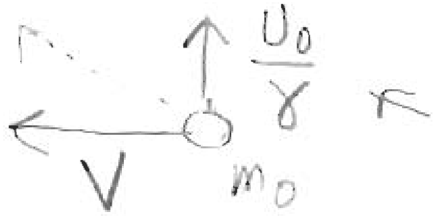
lately, the mass (was rest mass)

Find $m(v)$

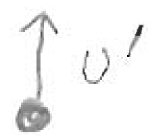
Collision between equal rest masses

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

before

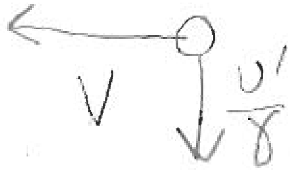


$$w = \sqrt{\left(\frac{u_0}{\gamma}\right)^2 + V^2}$$



$$w' = \sqrt{\left(\frac{u'}{\gamma}\right)^2 + V^2}$$

after



Horizontal

$$m(w)V = m(w')V$$

$$m(w) = m(w')$$

$$\rightarrow \quad w' = w$$

$$\rightarrow \quad u' = u_0$$

Vertical

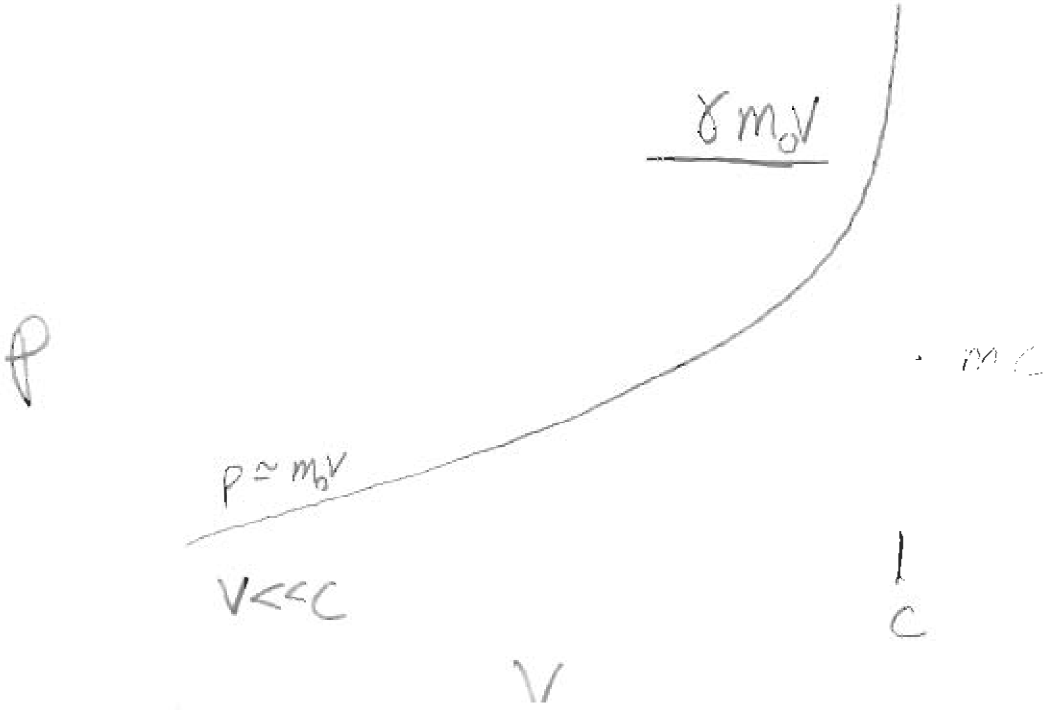
$$-m(u_0)u_0 + m(w)\frac{u_0}{\gamma} = +m(u_0)u_0 - m(w)\frac{u_0}{\gamma}$$

$$2m(u_0)u_0 = \frac{2m(w)}{\gamma}u_0$$

$$m(w) = \gamma m(u_0)$$

$$\lim_{u_0 \to 0} w = V \qquad \lim_{u_0 \to 0} m(u_0) = m(0) = m_0$$

$$m(v) = \gamma m_0 = \frac{m}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$



Energy   Non-relativistic

$$K_b - K_a = \int_a^b \vec{F} \cdot d\vec{r} \qquad \text{1-d}$$

$$= \int_a^b \frac{dp}{dt}\, dx \qquad dx = v\,dt$$

$$\frac{dp}{dt} = \frac{d}{dt}(m_0 v)$$

$$= m_0 \frac{dv}{dt}$$

non-relativistic

$$K_b - K_a = m_0 \int_a^b \frac{dv}{dt} v \, dt = \frac{1}{2} m_0 \int_a^b \frac{d}{dt}(v^2)\, dt$$

$$K_b - K_a = \frac{1}{2} m_0 \left(v_b^2 - v_a^2\right)$$

<u>Relativistic</u> (1-dim)

$$\frac{dp}{dt} \neq m_0 \frac{dv}{dt} \, , \; = \frac{d}{dt}\left(m(v)\, v\right)$$

<u>still</u>

$$K_b - K_a = \int_a^b \frac{dp}{dt}\, dx = \int_a^b \underbrace{\frac{d}{dt}\left(m(v) v\right) v\, dt}_{\text{integrate by parts!}}$$

$$= m(v) v^2 \Big|_a^b - \int_a^b m(v) v \frac{dv}{dt}\, dt$$

$$m(v) = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\int \frac{m_0 v \frac{dv}{dt}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\, dt = -m_0 c^2 \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$K_b - K_a = \frac{m_0 v^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} + m_0 c^2 \sqrt{1-\left(\frac{v}{c}\right)^2}$$

point a choose $v = 0$