UCSB | Physics 2c

Doppler Effects

Familiar from sirens for

For light situation more sub

Sound. A above middle C $440\,Hz$

→ something thumping 440 times per second..

Human Ear: responds to frequency ν, ν_0 at source

$$20\,Hz < \nu < 20{,}000\,Hz$$

2 kHz best.

8-20 kHz disappears with old age

Speed of Sound: $w \simeq 330 \frac{m}{s}$ in air

Recall: $\omega = 2\pi\nu$, $\underbrace{\omega_0 = 2\pi\nu_0}_{\text{at source}}$

$$T_0 = T = \frac{1}{\nu}$$

Sound is a <u>compression</u> wave.

molecules compressed

"rarified"

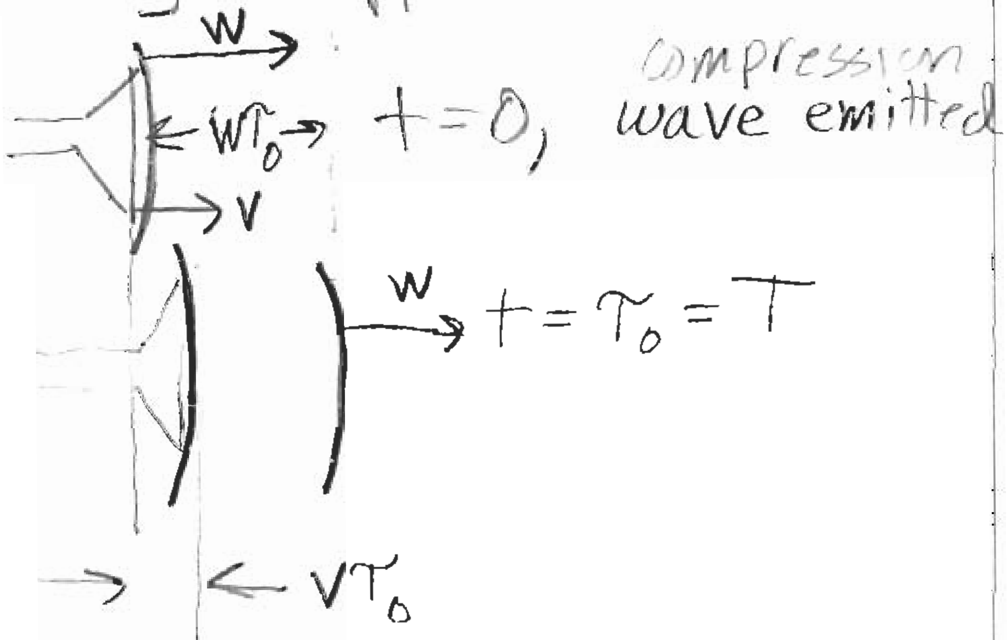
what is heard

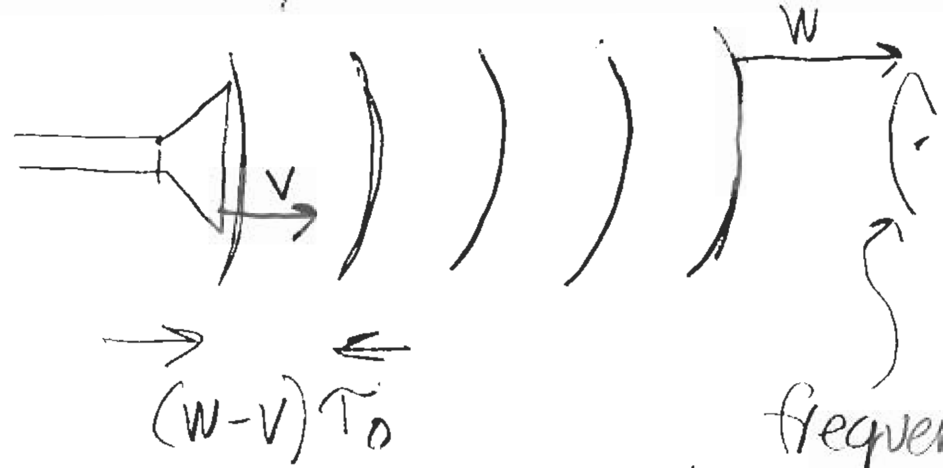
→ wavelength $L = wT_0$

$$\nu = \frac{1}{T_0} = \frac{w}{L}$$

Sound From a Source Moving Toward You (Non Relativistic)

→ frequency goes up
→ wavelength appears reduced.





frequency heard <u>higher</u>

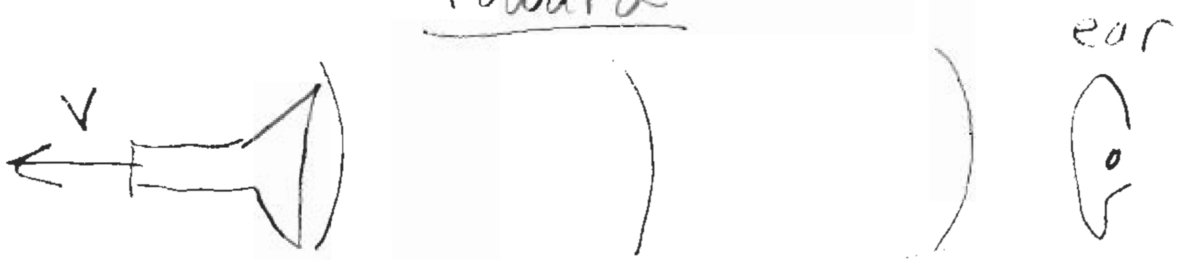
time between compressions perceived by ear

$$W\tau = (W-V)\tau_0$$

$$\nu_D = \frac{1}{\tau} = \frac{W}{W-V}\cdot\frac{1}{\tau_0}$$

$$\boxed{\nu_D = \frac{1}{1-\frac{V}{W}}\,\nu_0}$$

<u>toward</u>



$$\nu_D = \frac{1}{1+\frac{V}{W}}\,\nu_0$$

<u>away</u>

Example
Ambulance

$v = 60$ miles/hour

$v = 26$ meters/second

$\nu_0 = 1 \text{ kHz} = 1000 \text{ Hz}$ ($\simeq$ B5)

$w = 330$ meters/second.

Toward $\quad \nu_D = \dfrac{1000}{1 - \frac{26}{330}} = 1085.5 \text{ Hz}$

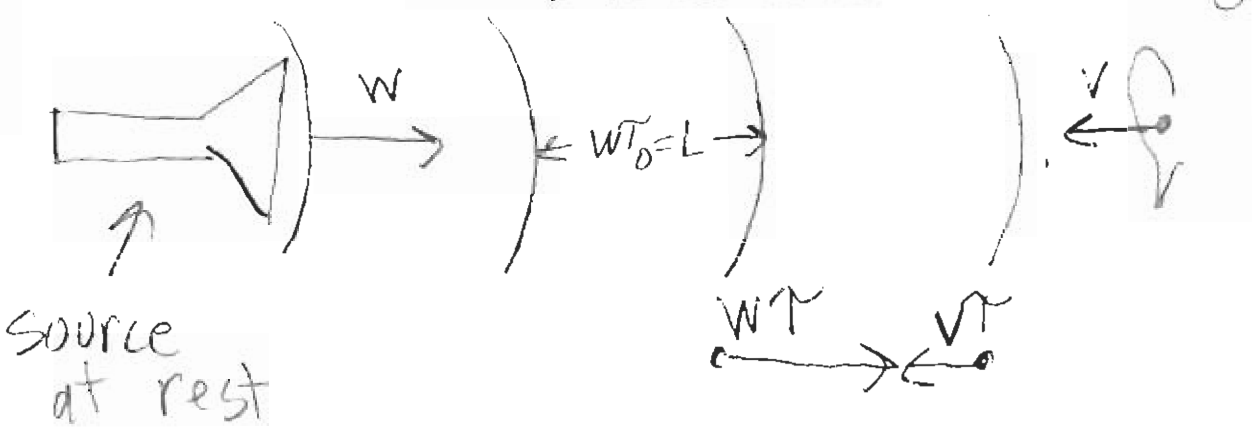
$\simeq$ C#6

~ whole tone

Away $\quad \nu_D = \dfrac{1000}{1 + \frac{26}{330}} = 927.0 \text{ Hz}$

~ Bb5

What if the detector is moving?



$$wT + vT = wT_0$$

$$T = \frac{w}{w+v} T_0$$

$$\nu = \frac{1}{T} = \left(1 + \frac{v}{w}\right) \nu$$

60 mph toward the ambulance

$$\nu_D = \left(1 + \frac{26}{330}\right) \cdot 1000 \text{ Hz}$$

$$\nu_D = \underline{1078.8} \text{ Hz} \neq 1085.5 \text{ Hz}$$

not same as source coming toward

Running Away From Soorce

$$\nu_D = \frac{1}{\tau} = \left(1 - \frac{v}{w}\right)\nu_0$$

ambulance $\nu_D = \left(1 - \frac{26}{330}\right) 1000 \text{ Hz}$

$$\nu_D = 921.2 \text{ Hz} \neq 927.0 \text{ Hz}$$

Violation of Relativity

can tell which is moving, source or detector. Why?

→ sound moves in a medium

→ light does not ⇒ different doppler

Light: Frequency causes color

ν_0: much higher than sound frequency.

Green Light: $L = 500\ \text{nm} = \frac{c}{\nu_0}$

$$\nu_0 = \frac{30 \cdot \frac{10^{-2}}{10^{-9}}\ \frac{m}{s}}{500 \cdot 10^{-9}\ m}$$

$$= \frac{3}{50}\ 10^7 \cdot 10^9\ \frac{1}{s}$$

$$\nu_0 = 6 \cdot 10^{14}\ Hz$$

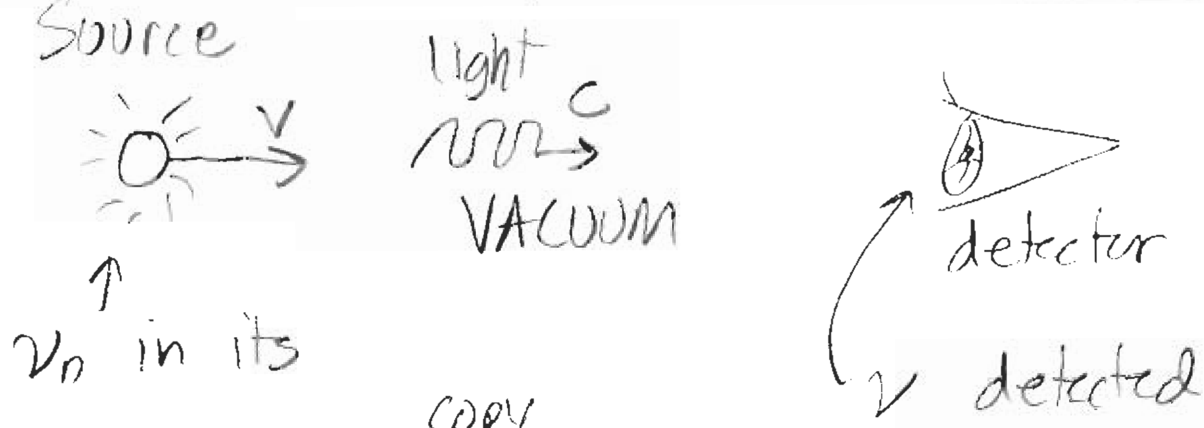
Green Light $\nu_0 = 600 \cdot 10^{12}\ Hz = 600\ THz$

Visible: ν_0 450-700 THz] So high, L used

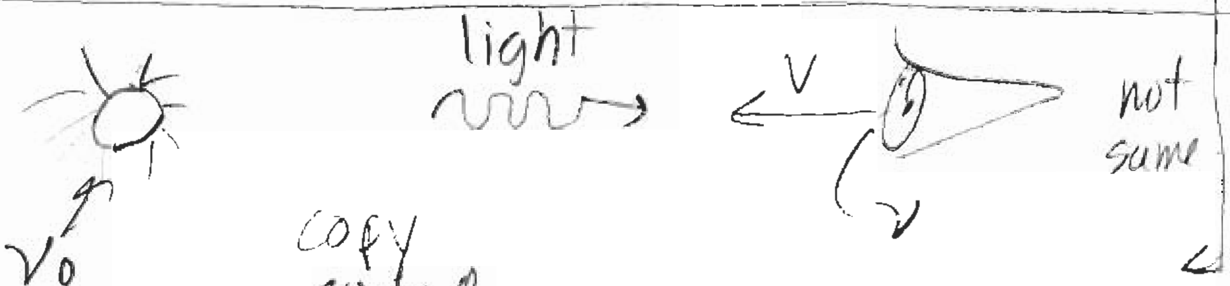
$L = 380\text{-}780$ nm

Radio AM 520-1610 kHz
FM 87-108 MHz

TV: Ch 1-6 44 → 88 MHz
7-13 174-216 MHz
14-83 471-885 MHz



$$\xrightarrow[\text{(wrong)}]{\substack{\text{copy}\\ \text{sound}}} \quad \nu = \frac{\nu_0}{1 - v/c}$$



$$\xrightarrow{\substack{\text{copy}\\ \text{sound}}} \quad \nu = \left(1 + \frac{v}{c}\right)\nu_0$$

Answer.

$$\nu = \sqrt{\frac{1 + v/c}{1 - v/c}}\;\nu_0$$

a little bit of both.

Derivation: $\neq c\tau_0 \neq c\tau$ source moving

$(c - v)\tau = L$

$\nu_0 = \frac{1}{\tau_0}$

time dilitation

$\tau = \gamma \tau_0$

$$c\tau_D = (c - v)\tau$$

$$\tau_D = \left(1 - \frac{v}{c}\right)\tau$$

$$\nu_D = \frac{1}{\tau_D} = \frac{1}{1-\frac{v}{c}}\frac{1}{\tau} = \frac{1}{1-\frac{v}{c}}\frac{1}{\gamma}\frac{1}{\tau_0}$$

$$\nu_D = \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}}\nu_0$$

$$= \frac{\sqrt{\left(1+\left(\frac{v}{c}\right)\right)\left(1-\left(\frac{v}{c}\right)\right)}}{1-\frac{v}{c}}$$

$$\boxed{\nu_D = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,\nu_0}$$

frequency

coming toward

$$L_D = \frac{c}{\nu_D} = \sqrt{\frac{1-v/c}{1+v/c}}\frac{c}{\nu_0}$$

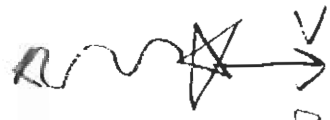
$$\boxed{L_D = \sqrt{\frac{1-v/c}{1+v/c}}\,L_0}$$

$$\nu_D = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\,\nu_0 \qquad L_D = \sqrt{\frac{1+v/c}{1-v/c}}\,L_0$$

moving away

Red Shift

Star



$\beta = \frac{v}{c}$

far away stars mostly do this

L_D

detect 700 nm

$L_0 = 400\text{ nm}$

$$L_D = L_0 \cdot \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$1 + z \equiv \frac{L_D}{L_0} = \sqrt{\frac{1 + v/c}{1 - v/c}}$$

redshift definition

$$(1+z)^2 = \left(\frac{L_D}{L_0}\right)^2 = \frac{1 + \frac{v}{c}}{1 - v/c}$$

$$\left(\frac{L_D}{L_0}\right)^2 - \frac{v}{c}\left(\frac{L_D}{L_0}\right)^2 = 1 + \frac{v}{c}$$

$$\left(\frac{L_D}{L_0}\right)^2 - 1 = \frac{v}{c}\left(1 + \left(\frac{L_D}{L_0}\right)^2\right)$$

$$\beta = \frac{v}{c} = \frac{\left(\frac{L_D}{L_0}\right)^2 - 1}{\left(\frac{L_D}{L_0}\right)^2 + 1}$$

$$\beta = \frac{\left(\frac{7}{4}\right)^2 - 1}{\left(\frac{7}{4}\right)^2 + 1} = 0.51$$