Views of Various Event Pairs

(1) Length Contraction

(2) Time Dilitation

(1) Length Contraction:

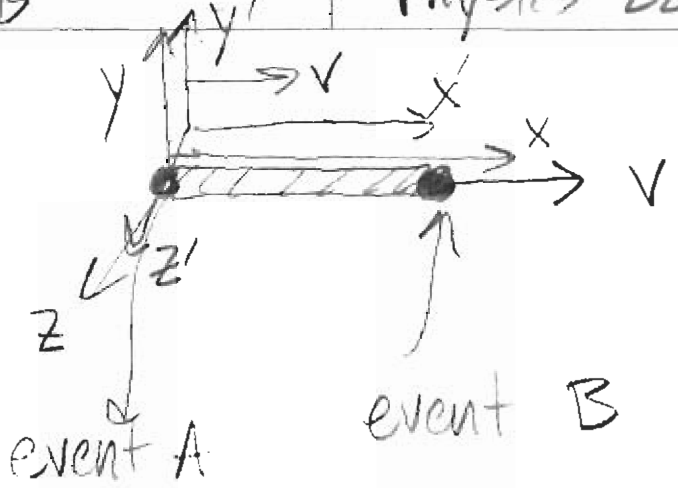
l_0 = length of rod in its own reference frame

If you are in a frame where it looks like the rod is moving at speed v $\parallel$ to its length, your measured length will be contracted!

$$l = \frac{l_0}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \frac{v}{c}$$

Key point: How do you measure the length of a moving rod?

→ Two simultaneous events in your frame, at ends of the rod ... (will not look simultaneous in the rod's rest frame)



stick at rest in pr

$t_A = 0$ $\qquad$ $t_B = 0$

$x_A = 0$ $\qquad$ $x_B = $ unknown

but $\quad x_A' = \gamma(x_A - \beta c \underbrace{t_A}_{\to 0}) = \gamma x_A = 0$

$$x_B' = \gamma(x_B - \beta c \underbrace{t_B}_{\to 0}) = \gamma x_B$$

$$\underbrace{x_B' - x_A' = l_0}_{\text{in rest frame}} = \gamma(x_B - x_A) = \gamma x_B$$

$$x_B = \frac{l_0}{\gamma}$$

Question: t_A', t_B' ?

$$ct_A' = \gamma(ct_A - \beta x_A) = 0$$

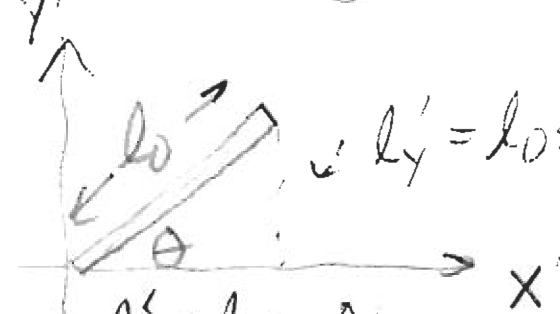
$$ct_B' = \gamma(ct_B - \beta x_B) = \gamma\left(0 - \beta \frac{l_0}{\gamma}\right)$$

$$\boxed{t_B' = -\beta \frac{l_0}{c}}$$

TOPS

Orientation of Moving Rod

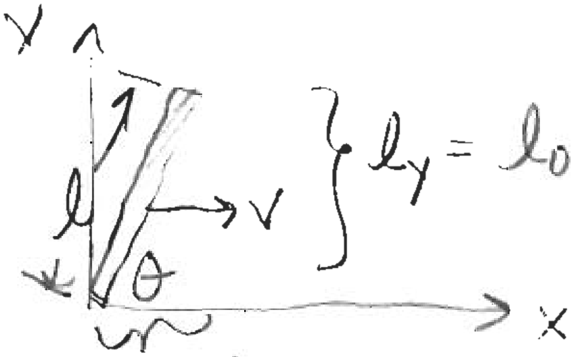
in x', y':



$$l'_y = l_0 \sin\theta_0$$

$$l'_x = l_0 \cos\theta_0$$

in x, y



$$l_y = l_0 \sin\theta_0$$

$$l_x = \frac{l_0}{\gamma}\cos\theta_0$$

$$\tan\theta = \frac{l_y}{l_x} = \frac{l_0 \sin\theta_0}{\frac{l_0}{\gamma}\cos\theta} = \gamma \tan\theta_0$$

$$\theta = \tan^{-1}(\gamma \tan\theta)$$

$$l = \sqrt{\left(\frac{l_0}{\gamma}\right)^2 \cos^2\theta + l_0^2 \sin^2\theta}$$

$$= l_0 \sqrt{(1-\beta^2)\cos^2\theta_0 + \sin^2\theta_0}$$

$$\frac{1}{\gamma^2}$$

$$l = l_0 \cdot \sqrt{1 - \beta^2 \cos^2\theta_0}$$

(2) Time Dilitation

Two events at the same point in space in moving frame, happening at different times.

$$x_A' = 0 = x_B'$$

$$t_A' = 0 \qquad t_B' = \tau$$

$$ct_A = \gamma(ct_A' + \beta x_A')$$

$$ct_A = 0 \qquad t_A = 0$$

$$ct_B = \gamma(ct_B' + \beta x_B')$$

$$ct_B = \gamma c \tau$$

$$\boxed{t_B = \gamma \tau} \leftarrow \gamma > 0$$

$$x_A = \gamma(\underbrace{x_A'}_{0} + \beta c \underbrace{t_A'}_{0}) = 0$$

$$x_B = \gamma(x_B' + \beta c t_B')$$

$$\boxed{x_B = \gamma \beta c \tau}$$

proton (from supernova

smash!

muon.

top of atmosphere

30,000 ft
$\simeq 10,000\,m = 10\,km$

$R_e = 6400\,km$

atm. $\frac{1}{640}$

Earth

lifetime of muon?

$\tau \simeq 2 \cdot 10^{-6}$ seconds

can it get to earth's surface?

First Guess

$c\tau = 30\,\frac{cm}{ns} \cdot 2000\,ns$

$= 60,000\,cm$

$c\tau = 600$ meters

$\ll 10,000$ meters

But LOTS of muons at earth's surface.. raining down

answer

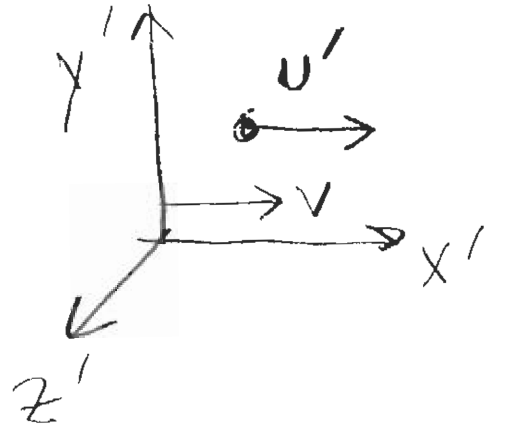
$\gamma\beta c\tau \sim 10,000$

$\gamma\beta \simeq \frac{10,000}{600} \simeq 16 = \frac{\beta}{\sqrt{1-\beta^2}}$

$\beta \simeq \frac{16^2}{\ldots} \simeq \frac{256}{\ldots} \sim .996$

Addition of velocities

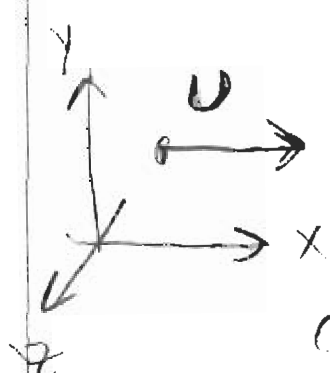
(different than book a bit)



$$\Delta x' = u' \Delta t'$$

$$\Delta x' = c\beta_u' \, \Delta t'$$

$$\beta_u' \equiv \frac{u}{c}$$



$$\Delta x = \gamma(\Delta x' + \beta_v c \Delta t')$$

$$\beta_v = \frac{v}{c}$$

$$c\Delta t = \gamma(c\Delta t' + \beta_v \Delta x')$$

$$u = \frac{\Delta x}{\Delta t} = \frac{\gamma(\Delta x' + \beta_v c \Delta t')}{\frac{1}{c}\gamma(c\Delta t' + \beta_v \Delta x')}$$

$$= \frac{\gamma \Delta t'\left(\left(\frac{\Delta x'}{\Delta t'}\right) + c\beta_v\right)}{\gamma \Delta t'\left(c + \beta_v \frac{\Delta x'}{\Delta t'}\right)}$$

$$u = \frac{c\beta_u' + c\beta_v}{\frac{1}{c}(c + \beta_v c \beta_u')}$$

$$\boxed{\beta_u = \frac{u}{c} = \frac{\beta_v + \beta_u'}{1 + \beta_v \beta_u'}}$$

<u>example</u> $\quad u' = 0.98\,c = (1-\delta_u')c$

$$v = 0.99\,c = (1-\delta_v)c$$

$$\beta_u = 1-\delta_u = \frac{1-\delta_v + 1-\delta_u'}{1+(1-\delta_v)(1-\delta_u')}$$

$$= \frac{2-(\delta_v+\delta_u')}{2-(\delta_v+\delta_u')+\delta_v\delta_u'}$$

$$\cong \frac{1}{1+\dfrac{\delta_v\delta_u'}{2-(\cancel{\delta_v+\delta_u'})}}$$

$\rightarrow$ higher order

$$\beta_u \simeq \frac{1}{1+\frac{1}{2}\delta_v\delta_u'}$$

$$\beta_u \simeq 1-\frac{1}{2}\delta_v\delta_u'$$

$$\simeq 1-\frac{1}{2}(10^{-2})(2\cdot10^{-2})$$

$$\beta_u \simeq 1-10^{-4} \simeq 0.9999000$$

<u>exact</u> $\quad 0.999898$