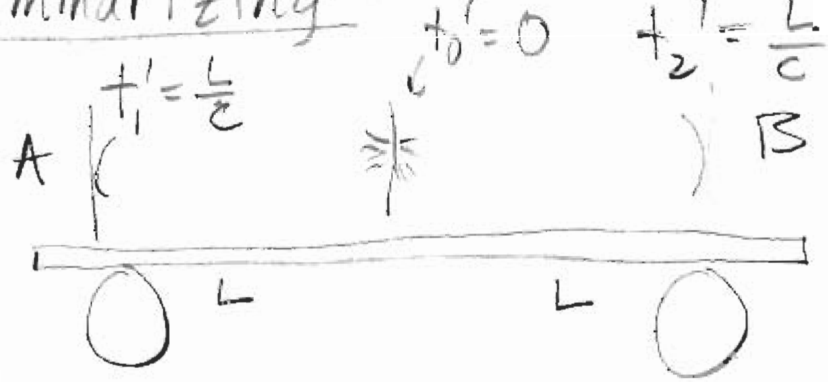
Summarizing



train frame

events A + B simultaneous

ground frame

$$t_1 = \sqrt{\frac{1+\beta}{1-\beta}}\,\frac{L}{c} = \frac{L/\gamma}{c(1+\beta)} < t_2 = \sqrt{\frac{1-\beta}{1+\beta}}\,\frac{L}{c} = \frac{L/\gamma}{c(1-\beta)}$$

events A^* + B^* <u>not</u> simultaneous.

Timelike + Spacelike Intervals

two events: x_A, t_A $\qquad$ x_B, t_A

$$L \equiv x_B - x_A$$

$$T = t_B - t_A$$

look at: $\dfrac{L}{T}$

- $> c \rightarrow$ can find another frame where events simultaneous. Call sep is space, spacelike
- $= c$
- $< c \rightarrow$ can find another frame where events

$$x_A' = \gamma(x_A - \beta c t_A)$$

$$ct_A' = \gamma(ct_A - \beta x_A)$$

$$x_B' = \gamma(x_B - \beta c t_B)$$

$$ct_B' = \gamma(ct_B - \beta x_B)$$

$$L' = x_B' - x_A' = \gamma(x_B - x_A - \beta c(t_B - t_A))$$

$$L' = \gamma(L - \beta c T) = \gamma L\left(1 - \frac{\beta c}{(L/T)}\right)$$

$$cT' = \gamma(cT - \beta L) = \gamma c T\left(1 - \beta\frac{(L/T)}{c}\right)$$

1 $L/T > c$ $\quad \frac{\beta c}{(L/T)} < \beta < 1$

$$1 - \frac{\beta c}{(L/T)} > 1 - \beta > 0$$

$\underline{L' > 0}$ "space like"

but now look at T'

$$\frac{(L/T)}{c} > 1$$

can make $cT' = 0$!

$$cT' = \gamma c T\left(1 - \beta\frac{(L/T)}{c}\right) = 0$$

$$\uparrow$$

$$\beta = \frac{c}{(L/T)} < 1$$

In this frame, events look simultaneous! But separated by a non-zero distance.

$\boxed{\text{spacelike separation}} \longleftrightarrow \boxed{\dfrac{L}{T} = \dfrac{\Delta x}{\Delta t} > c}$

$\boxed{\begin{matrix} L' > 0 \text{ always} \\ T' \text{ any sign} \end{matrix}} \longleftrightarrow \boxed{\begin{matrix} \exists \text{ frame where} \\ T' = 0 \\ L' > 0 \end{matrix}}$

$\updownarrow$

$\boxed{\text{event A cannot possibly influence event B}}$

(2) $\dfrac{L}{T} = c$

$$L' = \gamma L\left(1 - \frac{\beta c}{\underbrace{(L/T)}_{=c}}\right) \qquad \gamma L(1-\beta)$$

$$cT' = \gamma cT\left(1 - \beta \frac{\overbrace{(L/T)}^{=c}}{c}\right) = \gamma cT(1-\beta)$$

$$\frac{L'}{T'} = \frac{L}{T}\frac{(1-\beta)}{(1-\beta)} = \frac{L}{T} = c$$

→ something moving at the speed of light does so in all frames

$L' > 0 \qquad T' > 0$ always

"on the light cone"

(3) $L/T < c$

$$L' = \gamma L\left(1 - \frac{\beta c}{(L/T)}\right)$$

$\frac{c}{(L/T)} > 1$, β big enough, $1 - \beta\frac{c}{(L/T)} < 0$!

or, $L' = 0 = \gamma L\left(1 - \frac{\beta c}{L/T}\right)$

$$\beta = \frac{L/T}{c} < 1$$

In frame with this β, $L' = 0$!

$$cT' = \gamma cT\left(1 - \beta\frac{(L/T)}{c}\right)$$

always less than one

$$cT' > 0$$

timelike separation $\Leftrightarrow$ $\frac{L}{T} = \frac{\Delta x}{\Delta t} < c$

L' any sign
$T' > 0$ always

$\exists$ frame where $L' = 0$, $T' > 0$

$\Updownarrow$

event A can influence event B

Quick Test

"interval" Δs^2

$$\Delta s^2 \equiv (c\Delta t)^2 - (\Delta x)^2$$

$$= (cT)^2 - L^2 \quad \text{(original frame)}$$

$$\Delta s'^2 = (cT')^2 - L'^2$$

$$= \gamma^2(cT - \beta L)^2 - \gamma^2(L - \beta cT)^2$$

$$= \gamma^2 c^2 T^2 - \cancel{2\gamma^2 cT\beta L} + \gamma^2\beta^2 L^2$$

$$-\gamma^2 L^2 + \cancel{2\gamma^2 L\beta cT} - \gamma^2\beta^2 c^2 T^2$$

$$= \underbrace{\gamma^2(1-\beta^2)}_{1} c^2T^2 - \underbrace{\gamma^2(1-\beta^2)}_{1} L^2$$

$$\Delta s'^2 = (cT)^2 - L^2 = \Delta s^2$$

is a relativistic invariant

1) $\Delta s^2 > 0$ (lots of <u>time</u>) timelike

2) $\Delta s^2 = 0$ light cone

3) $\Delta s^2 < 0$ lots of <u>space</u> spacelike)