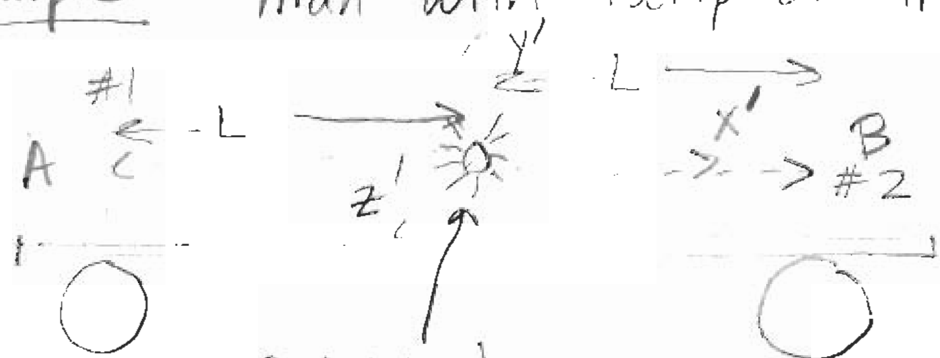
Use of Lorentz Transformations

Concept of "event": time now must be specified EXACTLY (well, to a small fraction of a nano second)

Example Man with lamp on train



first event... light switched on

coordinates? all zeroes if possible!

CHOICE: $x'_0 = y'_0 = z'_0 = t'_0 = 0$

next event(s)

#1 light pulse arrives at point A

$x'_1 = -L \quad y'_1 = 0 \quad z'_1 = 0$

$ct'_1 = L$ (note, positive)

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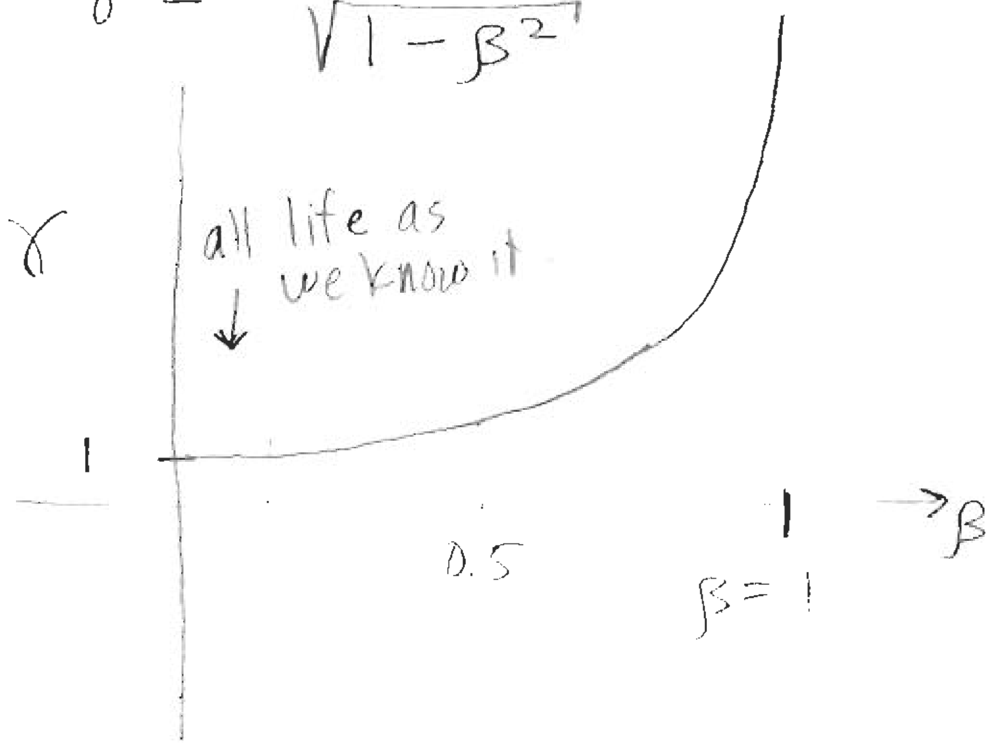
(2) $\gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}}$



<u>note</u>: for $\beta \ll 1$

$$\gamma \simeq (1-\beta^2)^{-1/2} \simeq \underbrace{1 + \tfrac{1}{2}\beta^2}_{\text{yes, related to kinetic energy}}$$

(3) use ct, ct' rather than t, t'

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma(-\beta x + ct)$$

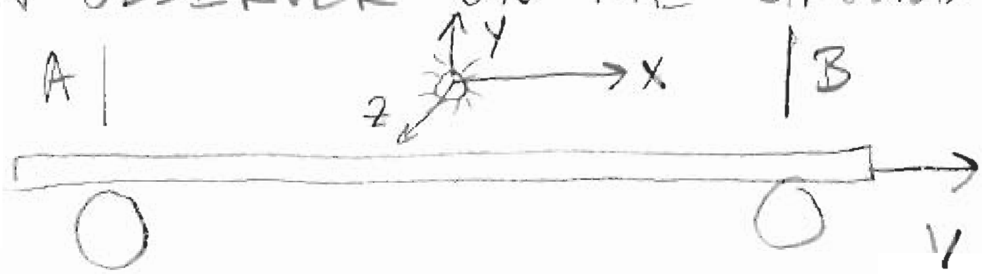
2 light pulse arrives at point B

$x_2' = +L \qquad y_2' = 0 \qquad z_2' = 0$

$ct_2' = L$

Notice: Events 1 + 2 are SIMULTANEOUS, in the frame of train car

WHAT DOES THIS LOOK LIKE TO AN OBSERVER ON THE GROUND?



First Choice: $x_0 = y_0 = z_0 = t_0 = 0$

now, viewed in this frame, A moves into light

First: A^* occurs early, B^* occurs late — exactly when?

A^*: Guess... $ct_1 + vt_1 \stackrel{?}{=} L$ (wrong!)

$t_1 = \frac{L}{c+v}$

$$ct_1 = \frac{L}{1 + \frac{v}{c}} \quad \text{(WRONG)}$$

B*
Guess for t_2

$$ct_2 - vt_2 = L$$

$$t_2 = \frac{L}{c - v}$$

$$ct_2 = \frac{L}{1 - v/c} \quad \text{(WRONG)}$$

Correct Way:

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma(-\beta x + ct)$$

invert

$$x = \gamma(x' + \beta ct')$$
$$y = y'$$
$$z = z'$$
$$ct = \gamma(\beta x' + ct')$$

#1:

$$x_1 = \gamma(-L + \beta L) = -\gamma(1-\beta)L$$
$$y_1 = y_1'$$
$$z_1 = z_1'$$
$$ct_1 = \gamma(\beta(-L) + L) = \gamma(1-\beta)L$$

$$ct_1 = \frac{1}{\sqrt{1-\beta^2}}(1-\beta)\ L$$

$$= \frac{1-\beta}{\sqrt{(1+\beta)(1-\beta)}}\ L$$

$$\boxed{ct_1 = \sqrt{\frac{1-\beta}{1+\beta}}\ L = \sqrt{\frac{1-v/c}{1+v/c}}\ L}$$

RIGHT

or $ct_1 = \sqrt{\frac{1-v/c}{1+v/c}}\ ct_1'$ (not $\frac{ct_1}{1+\frac{v}{c}}$)

$$\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \simeq \sqrt{1-\frac{v}{c}-\frac{v}{c}} = \sqrt{1-2\frac{v}{c}} \simeq 1-\frac{v}{c}$$

$$\frac{1}{1+v/c} \simeq 1-\frac{v}{c}$$

Agree to first order

$$ct_2 = \sqrt{\frac{1+\beta}{1-\beta}}\ L$$

note, in this frame, still true that $ct_1 < ct_2$

Note: $ct_1 = \sqrt{\frac{1-\beta}{1+\beta}}\ L = \sqrt{(1-\beta)(1+\beta)}\ \frac{L}{1+\beta}$

$$ct_1 = \sqrt{1-\beta^2}\,\frac{L}{1+\beta}$$

$$ct_1 = \frac{1}{\gamma}\,\frac{L}{1+\beta} \qquad\qquad \frac{L}{1+\beta}$$

$$ct_2 = \frac{1}{\gamma}\,\frac{L}{1-\beta} \qquad\qquad \frac{L}{1-\beta}$$

Right	Wrong

Lengths are always longer in the rest frame (here the train); they look shorter in other frames by a factor of $1/\gamma \neq 1$ by terms second order in $v/c = \beta$

STAEDTLER