Moving Observer

$$x' = ct - vt = (c-v)t$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$\boxed{c - v \neq c!}$

problem

Can't fix with y', z'.

Time what was it anyway?

Try a linear fix (non linear: unpleasant, accel.)

$$x' = Ax + Bt \qquad y' = y$$
$$t' = Cx + Dt \qquad z' = z$$

Use "thought experiments" to nail down A, B, C, D (Table 11.1)

→ origin of x' as viewed by x

$$x = vt \qquad x' = 0$$

$$0 = Avt + Bt \qquad \boxed{B = -Av}$$

→ origin of x viewed by x'
insist on velocity $-v$
(no way to distinguish frames based on speed of the other

$$x = 0 \qquad x' = -vt'$$

$$-vt' = A \cdot 0 + B \cdot t$$

$$-vt' = -Avt$$

$$-t' = -At$$

$$t' = C \cdot 0 + D \cdot t$$

$$At = Dt \qquad \boxed{D = A}$$

→ Look at that light beam

$$x = ct \qquad x' = ct'$$

$$x' = A(x - vt) \qquad (B = -Av)$$

$$t' = Cx + At$$

$$ct' = A(ct - vt)$$

$$t' = A\left(1 - \frac{v}{c}\right)t$$

$$A\left(1-\frac{v}{c}\right)t = Cct + At$$

$$-A\frac{v}{c^2}t = Ct$$

$$\boxed{C = -\frac{v}{c^2}\cdot A}$$

Summarize

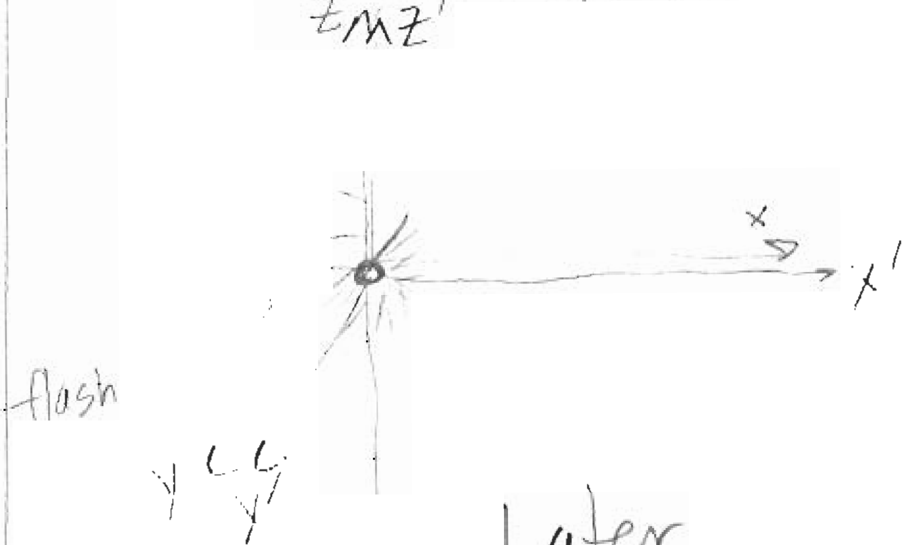
$$x' = A(x - vt)$$

$$t' = A\left(-\frac{v}{c}\frac{x}{c} + t\right)$$

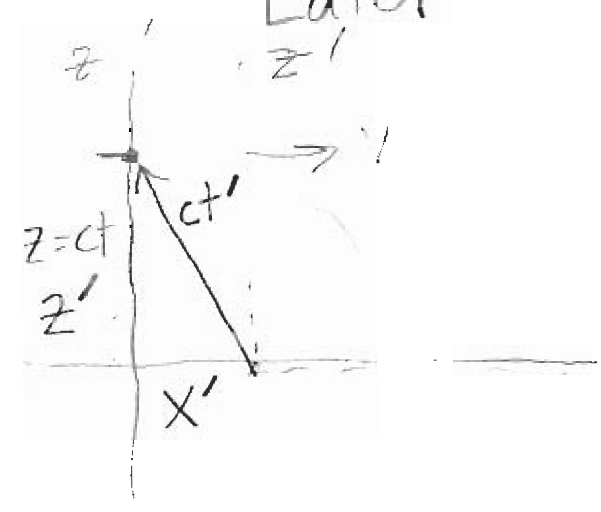
A still unknown. Not Busted Yet!

A → use second dimension, imagine circular light wave emitted

at: $x' = y' = t' = x = y = t = 0$



Later



y' $z = ct$

$x = 0$

$$x'^2 + z'^2 = c^2 t'^2$$

$$x' = A(x - vt)$$

$$z' = z = ct$$

$$t' = A\left(-\frac{v}{c}\frac{x}{c} + t\right)$$

$$A^2(-vt)^2 + c^2t^2 = c^2A^2[0 + t]^2$$

$$c^2\cancel{t^2} = c^2A^2t^2 - v^2A^2\cancel{t^2}$$

$$c^2 = A^2(c^2 - v^2)$$

$$A^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - (\frac{v}{c})^2}$$

$$A = + \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$$

(- root messes up for small time).

$$x' = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}\left(-\frac{v}{c}\frac{x}{c} + t\right)$$

The Lorentz Transformations

How we really use them..

(1) $\beta \equiv \frac{v}{c}$ (assume now in x-direction)

$-1 < \beta < 1$ cannot go faster than speed of light!