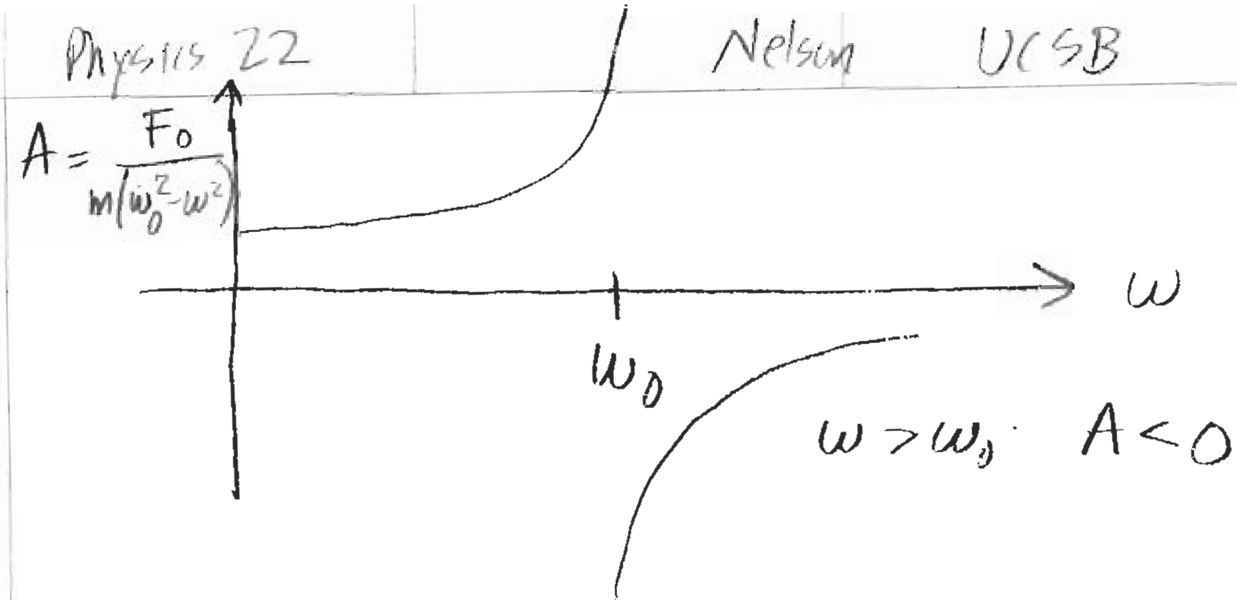


When the driving force has the same frequency as the natural frequency of the spring, the amplitude $|A| \to \infty$!!

Known as "resonance", physical origin is that none of the force's energy is wasted, all is "stored up"!

Damped

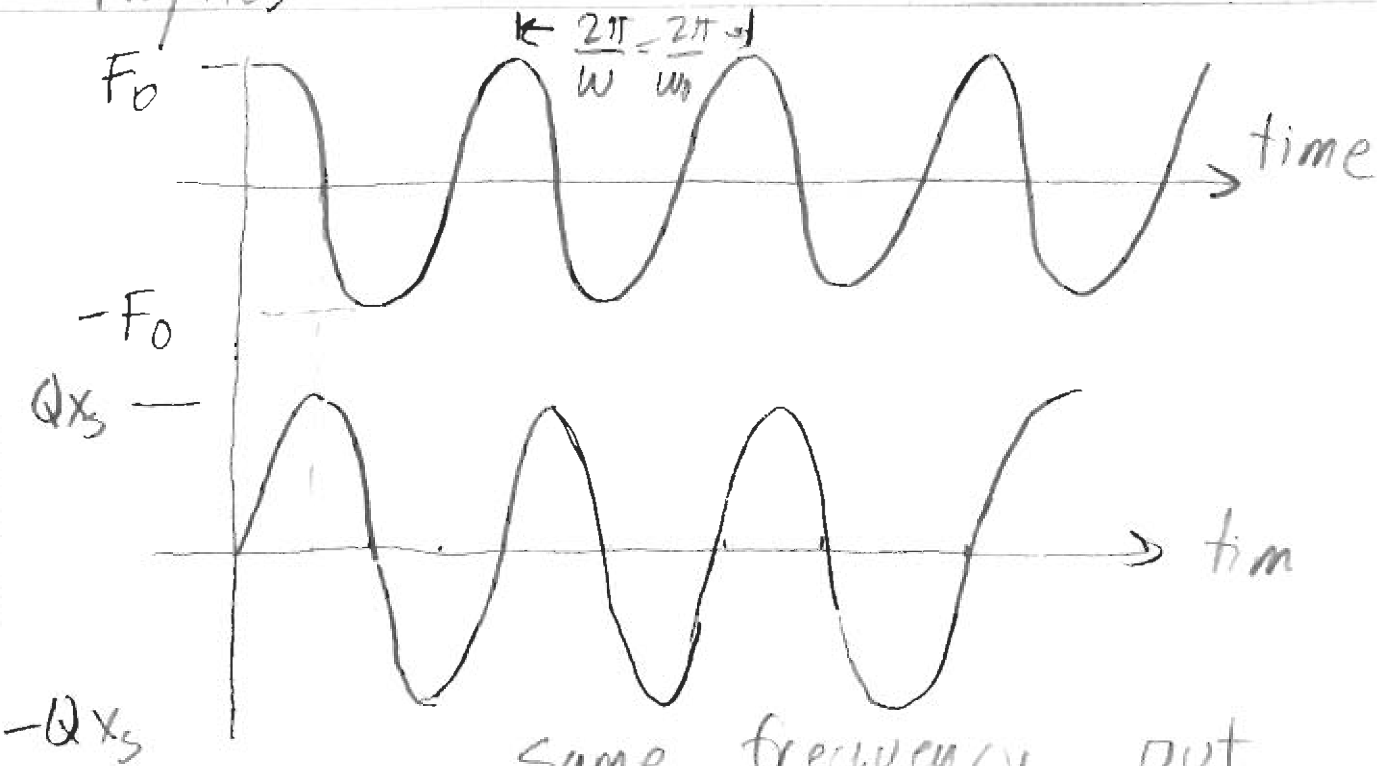
$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t = Re\left[\frac{F_0}{m} e^{i\omega t}\right]$$

$$z_H = e^{-(\gamma/2)t}\left(\alpha e^{i\omega_1 t} + \beta e^{-i\omega_1 t}\right)$$

$$z_F = A e^{i\omega t} \quad (\text{not } \omega_0 !)$$

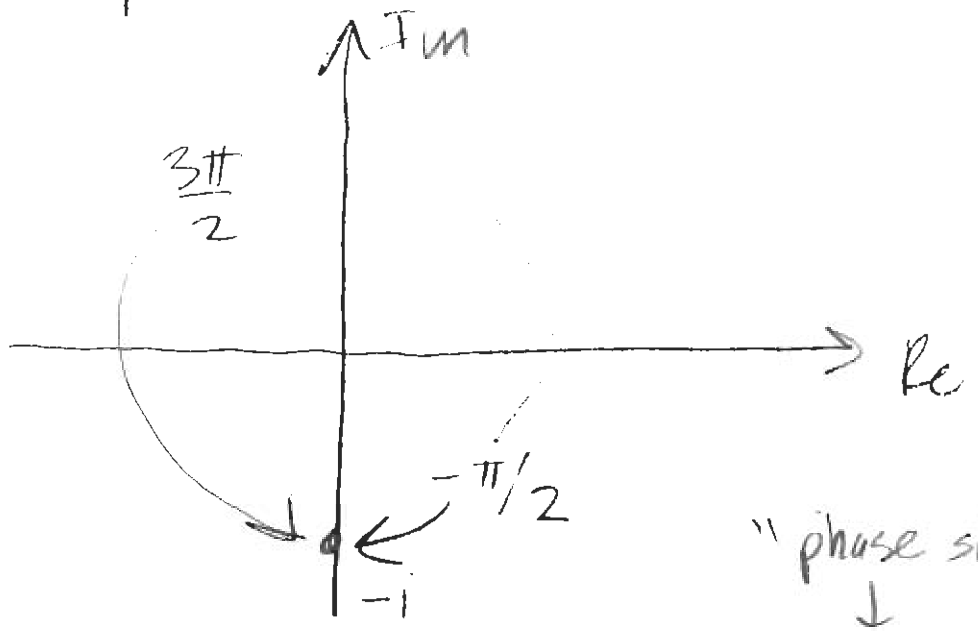
$$A\left[-\omega^2 + i\omega\gamma + \omega_0^2\right] e^{i\omega t} = \frac{F_0}{m} e^{i\omega t}$$

$$A = \frac{F_0}{m} \frac{1}{\omega_0^2 - \omega^2 + i\omega\gamma}$$



same frequency, out of phase

by: $\frac{1}{i} = -i = e^{i\frac{3}{2}\pi} = e^{-i\frac{1}{2}\pi}$



"phase shift"

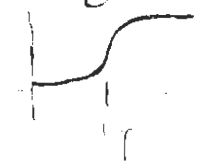
$$Re\left(\frac{x_s Q}{i} e^{i\omega t}\right) = Re\left(x_s Q\, e^{i(\omega t - \frac{\pi}{2})}\right)$$

$$\cos\left(\omega t - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - \omega t\right) = \sin \omega t$$

General Case

$$A = x_s \cdot \frac{1}{1 - r^2 + \frac{ir}{Q}} \qquad r = \frac{\omega}{\omega_0} \qquad Q = \frac{\omega_0}{\gamma}$$

$$= \rho e^{-i\phi} \qquad \tan\phi = \frac{\frac{r}{Q}}{1-r^2}$$



$$\rho^2 = x_s^2 \cdot \frac{1}{(1-r^2)^2 + \frac{r^2}{Q^2}}$$ ← magnitude, squared, of denominator

Near Resonance $r \cong 1$

$$r = \delta + 1, \quad \delta = r - 1$$

$$(1-r^2) = (1+r)(1-r) \simeq -(2+\delta)\delta$$

$$\simeq -2\delta$$

$$(1-r^2)^2 \simeq 4\delta^2$$

$$\frac{r^2}{Q^2} \simeq \frac{(\delta+1)^2}{Q^2} \simeq \frac{1}{Q^2} + \frac{2\delta}{Q^2} + \frac{\delta^2}{Q^2}$$

when $Q \gg 1$ → keep only that

$$A = \frac{F_0}{m} \frac{1}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

$\omega = \omega_0$ "Resonance"

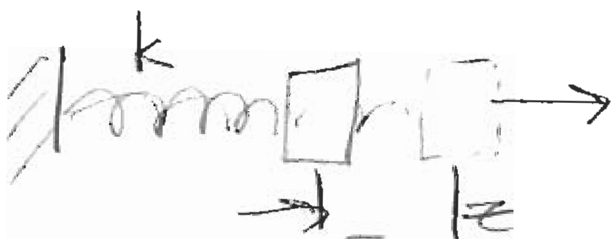
$$A = \frac{F_0}{m} \cdot \frac{1}{i\omega_0\gamma} \leftarrow$$ $\gamma \neq 0$ means A not infinite.

But A is imaginary... what does that mean??

<u>First...</u> recast expression

$\frac{F_0}{m}$ - what does it mean?

$F_0 \cdot \cos\omega t$
driving force



F_0, maximum

$$X_s = X_{static} = \frac{F_0}{k}$$

$$\omega_0^2 = \frac{k}{m}$$

$$k = m\omega_0^2$$

$$= \frac{F_0}{m\omega_0^2}$$

$$\frac{F_0}{m} = X_s \cdot \omega_0^2$$

$$A = X_s \cdot \frac{\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\gamma}$$