→ average over a cycle

→ assume $\frac{\gamma}{\omega_0} \ll 1$

↓

means... $e^{-\gamma t}$ ~ constant over one cycle of oscillator

$$\left\langle \frac{1}{2}kx^2 \right\rangle_1 \simeq \frac{1}{2}kA^2 e^{-\gamma t} \cdot \underbrace{\langle \cos^2(\omega_1 t+\phi)\rangle}_{\simeq 1/2}$$

one cycle avg

$$\left\langle \frac{1}{2}m\dot{x}^2 \right\rangle_1 \simeq \frac{1}{2}m\omega_1^2 A^2 e^{-\gamma t}\Big[\underbrace{\langle \sin^2(\omega_1 t+\phi)\rangle}_{\simeq 1/2}$$

$$+ \left(\frac{\gamma}{\omega_1}\right)\underbrace{\langle \sin(\omega_1 t+\phi)\cos(\omega_1 t+\phi)\rangle}_{\simeq 0}$$

$$+ \left(\frac{\gamma}{\omega_1}\right)^2 \langle \cos^2(\omega_1 t+\phi)\rangle \simeq \ \Big]$$

$$\left\langle \frac{1}{2} m \dot{x}^2 \right\rangle_1 \simeq \frac{1}{2} m \omega_1^2 A^2 e^{-\gamma t} \, \frac{1}{2}\left(1 + \left(\frac{\gamma}{\omega_1}\right)^2\right)$$

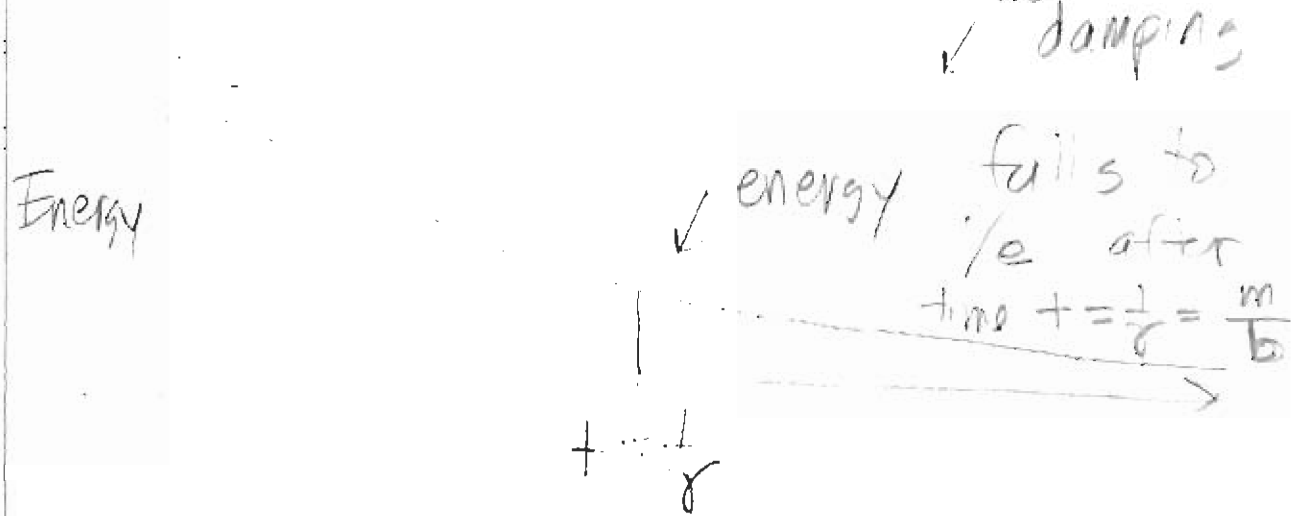
$$\omega_1^2 = \omega_0^2 \underbrace{\left(1 + \frac{1}{8}\frac{\gamma^2}{\omega_0^2}\right)}_{\simeq 1!} \qquad \left(\frac{\gamma}{\omega_1}\right)^2 \simeq 0!$$

$$\left\langle \frac{1}{2} m \dot{x}^2 \right\rangle_1 \simeq \frac{1}{4} m \omega_0^2 A^2 e^{-\gamma t}$$

only remnant of γ

$$E_1 = \left(\frac{1}{4} k A^2 + \frac{1}{4} m \omega_0^2 A^2\right) e^{-\gamma t}$$

$$E_1 = \frac{1}{2} k A^2 e^{-\gamma t}$$



STAEDTLER No 937 811E Engineer's Computation Pad

Driven Oscillators

Undamped

$$m\ddot{x} = -kx + F(t)$$

$$\ddot{x} + \omega_0^2 x = \frac{1}{m}F(t)$$

suppose $x_F(t)$ satisfies

$$\ddot{x}_F + \omega_0^2 x_F = \frac{1}{m}F(t)$$

suppose $x_H(t)$ satisfies

$$\ddot{x}_H + \omega_0^2 x_H = 0$$

"Homogenous"

consider

$$x_c \equiv x_F + x_H$$

$$\ddot{x}_c + \omega_0^2 x_c = \ddot{x}_F + x_H + \omega_0^2(x_F + x_H)$$

$$= \underbrace{\ddot{x}_F + \omega_0^2 x_F}_{\frac{1}{m}F(t)} + \underbrace{\ddot{x}_H + \omega_0^2 x_H}_{0}$$

$$\ddot{x}_c + \omega_0^2 x_c \text{ also } = \frac{1}{m}F(t)$$

we already know

$$x_H = r\cos(\omega_0 t + \phi) \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

Idea then

$x_H \rightarrow$ use it (and its adjustable parameters) to satisfy initial conditions

For study... choose $F(t) = F_0 \cos \omega t$
"driving term"

$$\ddot{x}_F + \omega_0^2 x_F = \frac{F_0}{m} \cos \omega t$$

$$\ddot{z}_F + \omega_0^2 z_F = \frac{F_0}{m} e^{i\omega t}$$

$$x_F = \mathrm{Re}(z_F)$$

Try $z_F = A e^{i\omega t}$

$$A(-\omega^2 + \omega_0^2) e^{i\omega t} = \frac{F_0}{m} e^{i\omega t}$$

$$A = \frac{F_0}{m(\omega_0^2 - \omega^2)} \leftarrow \text{NO FREE PARAMETERS}$$

not same

$$\boxed{x_c = \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega t + r \cos(\omega_0 t + \phi)}$$

free free

use initial conditions to obtain