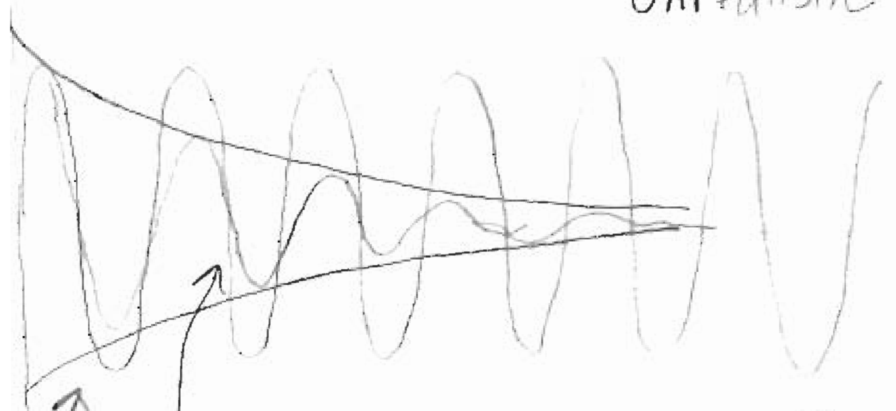
Damping

unrealistic! → friction etc



more realistic

exponential decay envelope

but no damping is exponential too! $e^{i\omega_0 t}$!

$$\cos(\omega_0 t) = \mathrm{Re}(e^{i\omega_0 t})$$

$$B\sin(\omega_0 t) + C\cos(\omega_0 t) = \mathrm{Re}(A e^{i\omega_0 t + i\phi})$$

$$m\ddot{x} = -kx - b\dot{x}$$

damping term

resists velocity

Try $x = \mathrm{Re}(z)$

complex

$z = z_0 e^{\alpha t}$ $\quad z_0, \alpha$ independent of time

z must satisfy same eq as x

$m\ddot{x} + b\dot{x} + kx = 0$ see p. 435 → 437

$m\ddot{z} + b\dot{z} + kz = 0$

$(m\alpha^2 + b\alpha + k) z_0 e^{\alpha t} = 0$

never zero in practice

$m\alpha^2 + b\alpha + k = 0$

quadratic! (think of b ...

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

$$\alpha = -\frac{1}{2}\frac{b}{m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

think → $b \to 0$

$$\alpha = \pm\sqrt{-\frac{k}{m}} = \pm i\omega_0$$

$$z = z_A e^{i\omega_0 t} + z_B e^{-i\omega_0 t}$$

$Re(z) = B\sin(\omega_0 t) + \ldots t)$

$\frac{1}{2i}(e^{i\omega_0 t} - e^{-i\omega_0 t})$

$$Re(z) = \left(\frac{B}{2i} + \frac{C}{2}\right) e^{i\omega_0 t} + \left(\frac{-B}{2i} + \frac{C}{2}\right) e^{-i\omega_0 t}$$

imaginary part works out

now .. $b \neq 0$ but $\boxed{\text{small} \ldots}$

$$\alpha = -\frac{1}{2}\frac{b}{m} \pm \sqrt{-\frac{k}{m} + \left(\frac{b}{2m}\right)^2}$$

$$\frac{b}{m} \equiv \gamma \qquad \frac{k}{m} = \omega_0^2$$

$$\alpha = -\frac{1}{2}\gamma \pm \sqrt{-\omega_0^2 + \frac{1}{4}\gamma^2}$$

shifts frequency a bit

to $i\omega_1 = \sqrt{-\omega_0^2 + \frac{1}{4}\gamma^2}$ $\quad \gamma \ll \omega_0$

a real term arises!

$$= \sqrt{-\omega_0^2}\left[1 - \frac{\gamma^2}{4\omega_0^2}\right]^{1/2} \simeq i\omega_0\left(1 - \frac{\gamma^2}{8\omega_0^2}\right) = i\omega_0 \quad i\left(\omega_0 - \frac{\gamma^2}{8\omega_0}\right)$$

$$Z = z_A e^{-\frac{1}{2}\gamma t + i\omega_1 t} + z_B e^{-\frac{1}{2}\gamma t - i\omega_1 t}$$

$$\boxed{x = Re(z) = A e^{-\frac{1}{2}\gamma t} \cos(\omega_1 t + \phi)}$$

the exponential envelope

Characterize oscillation cycles during Energy-loss time of $1/\gamma$ ...

→ Convention is to look at # of RADIANS advanced by oscillator

$\cos(\omega_0 t + \phi)$ → # Radians $= \omega_0 \cdot \frac{1}{\gamma}$

"Quality" of oscillator systems. → $\boxed{Q \equiv \frac{\omega_0}{\gamma}}$ $T = \frac{2}{\gamma}$

High Q → high quality
→ guitar string

$\frac{\omega_0 \left(\frac{2}{\gamma}\right)}{2\pi} = N$

$\frac{\omega_0}{\gamma}\frac{1}{\pi} = N$

$Q = \frac{\omega_0}{\gamma} = N\pi$

$N = \frac{Q}{2\pi}$ ← # oscillations
or,
$Q = 2\pi N$

$Q \gtrsim 10$ – equations above are OK

$Q \lesssim 10$ "poor oscillator"
→ you may want it!