$$B^2 + C^2 = A^2 \sin^2\phi + A^2 \cos^2\phi$$

$$B^2 + C^2 = A^2$$

$$A = \sqrt{B^2 + C^2}$$

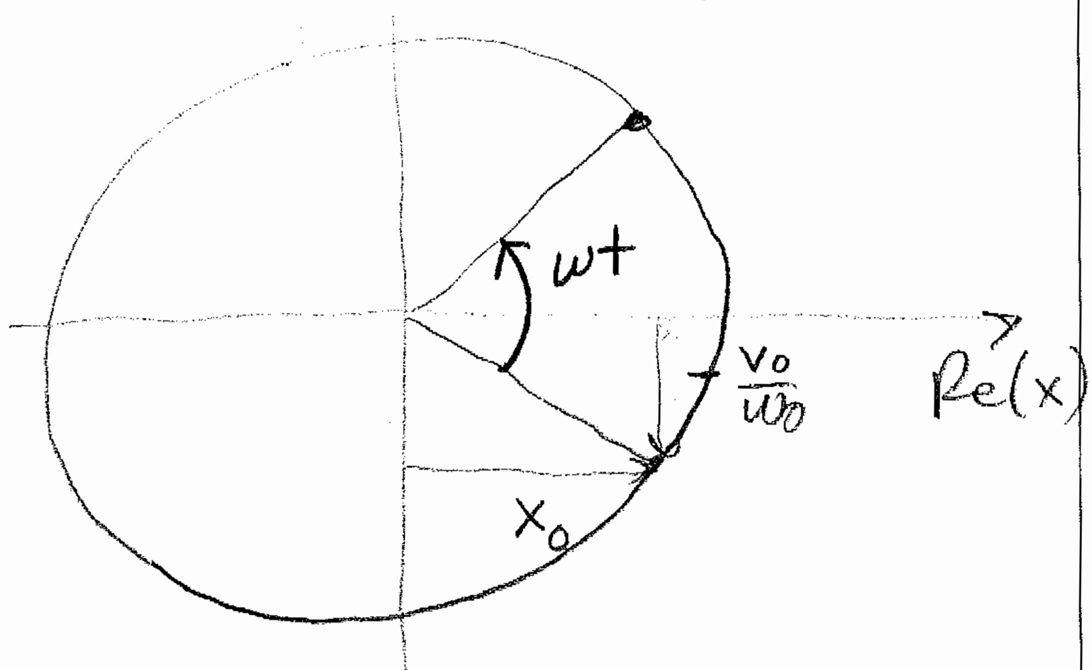
$$= \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2}$$

$$x(t) = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2} \cos(\omega_0 t + \phi)$$

$$\phi = \tan^{-1}\left(-\frac{v_0}{x_0 \omega_0}\right)$$

$$= \mathrm{Re}\left[\sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2}\, e^{+i\phi} e^{i\omega t}\right]$$

$\mathrm{Im}(x) \leftarrow$ no physical meaning



Comment about Energy

$$E = \text{constant (no friction)}$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$$

$$x = A\cos(\omega_0 t + \phi)$$

$$\dot{x} = -A\omega_0 \sin(\omega_0 t + \phi)$$

$$E = \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$$

but $\omega_0 = \sqrt{\frac{k}{m}}$ $\quad m\omega_0^2 = k$

$$= \frac{1}{2}kA^2\left(\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi)\right)$$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}k\left(x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2\right)$$

$$= \frac{1}{2}kx_0^2 + \frac{1}{2}\frac{k}{\omega_0^2}v_0^2 \qquad \frac{k}{\omega_0^2} = m$$

$$E = \underbrace{\frac{1}{2}kx_0^2}_{\text{initial PE}} + \underbrace{\frac{1}{2}mv_0^2}_{\text{initial KE}}$$

Not straightforward to use complex representation to get energy

Time Average Quantities

$$E = \underbrace{\tfrac{1}{2}kA^2\cos^2(\omega_0 t+\phi)}_{\substack{\text{instantaneous}\\ \text{potential energy}\\ U(t)}} + \underbrace{\tfrac{1}{2}kA^2\sin^2(\omega_0 t+\phi)}_{\substack{\text{instantaneous}\\ \text{kinetic energy}\\ K(t)}}$$

$$\langle U(t)\rangle = \frac{1}{T}\int_t^{t+T} U(t)\,dt \qquad T = \text{period}$$

$$= \tfrac{1}{2}kA^2\Big[\underbrace{\frac{1}{T}\int_t^{t+T}\cos^2(\omega_0 t+\phi)}_{\frac{1}{2},\ \text{independent of } \phi!}\Big]$$

$$\left.\begin{aligned}\langle U(t)\rangle &= \tfrac{1}{4}kA^2\\ \langle K(t)\rangle &= \tfrac{1}{4}kA^2\end{aligned}\right]\ \text{average energies equal}$$

(Simple Harmonic Oscillator)

generally

$$\langle K\rangle = \frac{n}{2}\langle U\rangle \qquad \left(\begin{array}{c}\text{Virial}\\ \text{theorem}\end{array}\right)$$

$U \propto r^n$

$n=2 \quad \langle K\rangle = \langle U\rangle \qquad n = -1$

$$\langle K\rangle = -\tfrac{1}{2}\langle U\rangle$$

more realistic

exponential decay envelope

but

no damping is exponential too! $e^{i\omega_0 t}$!

$$\cos(\omega_0 t) = \mathrm{Re}(e^{i\omega_0 t})$$

$$B\sin(\omega_0 t) + C\cos(\omega_0 t) = \mathrm{Re}(A e^{i\omega_0 t + i\phi})$$

$$m\ddot{x} = -kx - b\dot{x}$$

damping term

resists velocity

Try $\quad x = \mathrm{Re}(z)$

complex