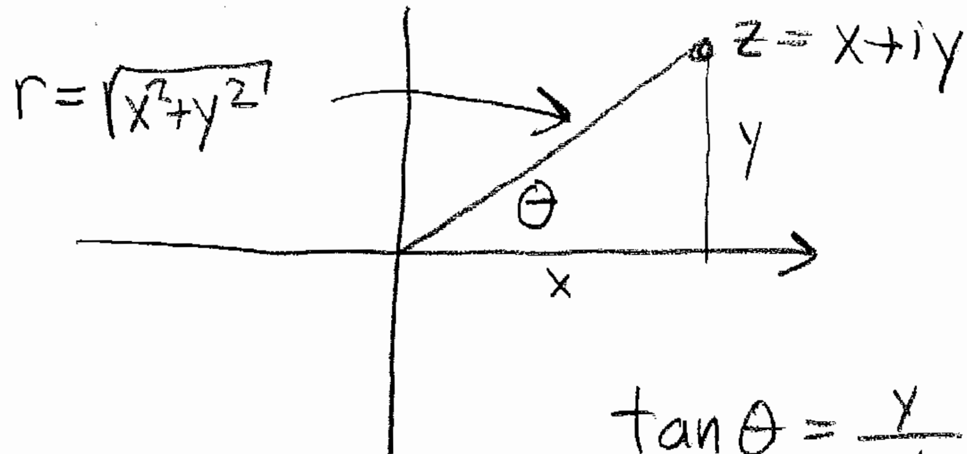
Polar Representation of Complex Numbers

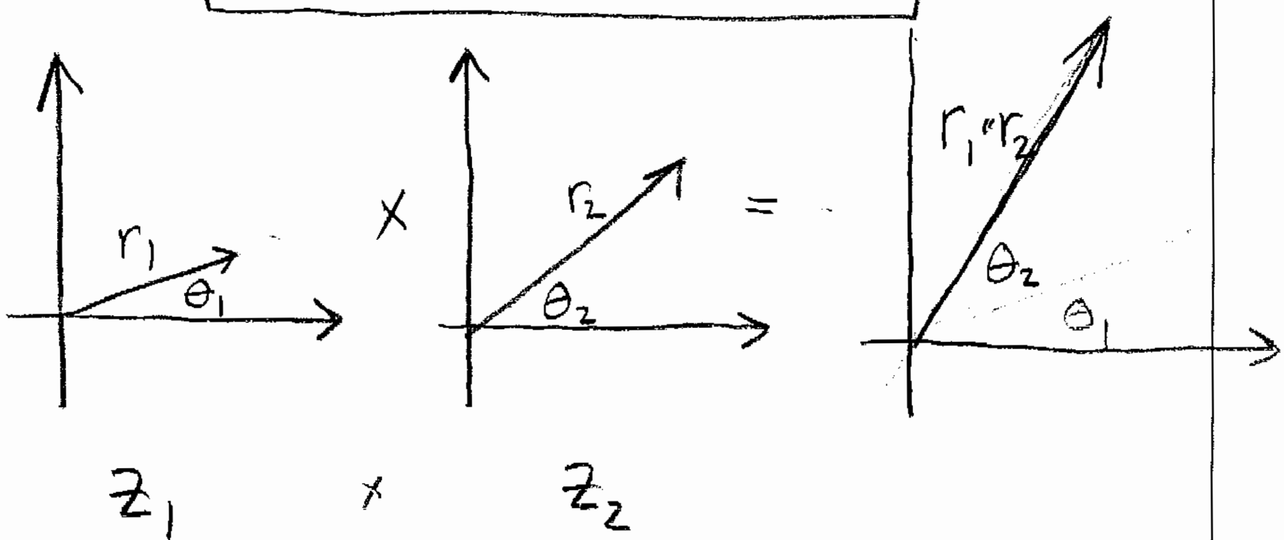


$$\tan\theta = \frac{y}{x}$$

$$z = x+iy = re^{i\theta}$$

$$z_1 = r_1 e^{i\theta_1} \qquad z_2 = r_2 e^{i\theta_2}$$

$$\boxed{z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}}$$



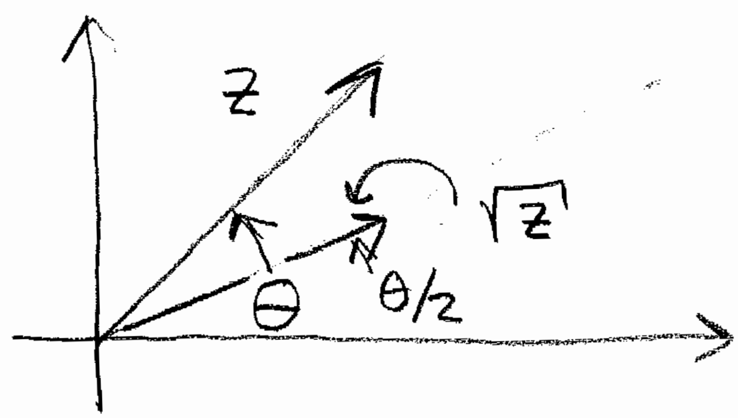
$$e^{2i\theta} = \cos 2\theta + i\sin 2\theta$$
$$= (\cos\theta + i\sin\theta)^2$$
$$= (\cos^2\theta - \sin^2\theta) + i\,2\sin\theta\cos\theta = \cos 2\theta + 2i\sin 2\theta$$

$$e^{3i\theta} = \cos 3\theta + i \sin 3\theta$$

$$\cos^3\theta - 3\sin^2\theta\cos\theta \qquad 3\cos^2\theta\sin\theta - \sin^3\theta$$

$$\sqrt{z} = \underbrace{\sqrt{x+iy}}_{\text{huh?}} = \sqrt{re^{i\theta}} = \sqrt{r}\,e^{i\frac{\theta}{2}}$$

$$= (x^2+y^2)^{1/4}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$$



$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

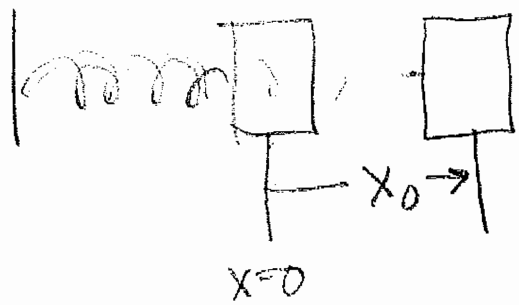
$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

$$\boxed{\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)} = Re(e^{i\theta}) = Re(e^{-i\theta})$$

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\boxed{\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)} = Im(e^{i\theta}) = -Im(e^{-i\theta})$$

SHO



release

$F = -kx$ $\qquad$ $\omega_0 = \sqrt{\frac{k}{m}}$

$m\ddot{x} = -kx$

$x = B\sin(\omega_0 t) + C\cos(\omega_0 t)$

B + C from <u>initial conditions</u>

<u>above</u>: <u>t = 0</u>, $x = x_0$, $\dot{x} = 0$

$x(0) = C$ $\qquad$ $\dot{x}(0) \Rightarrow$

$\boxed{C = x_0}$

$\dot{x} = B\omega_0\cos(\omega_0 t) - C\omega_0\sin(\omega_0 t)$

$\dot{x}(0) = B\omega_0 = 0$

$B = 0$

more generally,

$B = \frac{v_0}{\omega_0}$

$$\boxed{x = \left(\frac{v_0}{\omega_0}\right)\sin(\omega_0 t) + x_0\cos(\omega_0 t)}$$

$$x = A\cos(\omega_0 t + \phi)$$

$$= A\,\mathrm{Re}\left[e^{i(\omega_0 t + \phi)}\right]$$

$\tilde{A} = Ae^{i\phi}$

so useful.. $x = Ae^{i\omega_0 t + i\phi} = \tilde{A}e^{i\omega_0 t}$

(knowing real part of $e^{i\omega_0 t}$ implied!)

$$x = A\cos(\omega_0 t)\cos\phi - A\sin(\omega_0 t)\sin\phi$$

$$= (-A\sin\phi)\sin\omega_0 t + (A\cos\phi)\cos(\omega_0 t)$$

$$= \underset{\substack{\parallel \\ v_0/\omega_0}}{B}\,\sin(\omega_0 t) + \underset{\substack{\parallel \\ x_0}}{C}\,\cos(\omega_0 t)$$

$B = -A\sin\phi \qquad C = A\cos\phi$

$$\frac{B}{C} = \frac{-A\sin\phi}{A\cos\phi} = -\tan\phi$$

$$\boxed{\tan\phi = -\frac{B}{C} = -\frac{v_0}{x_0\,\omega_0}}$$

$v_0 = 0, \quad \phi = 0$

$x_0 = 0, \quad \phi = -\frac{\pi}{2}$ } $\cos(\omega_0 t - \frac{\pi}{2}) = \sin(\omega_0 t)$