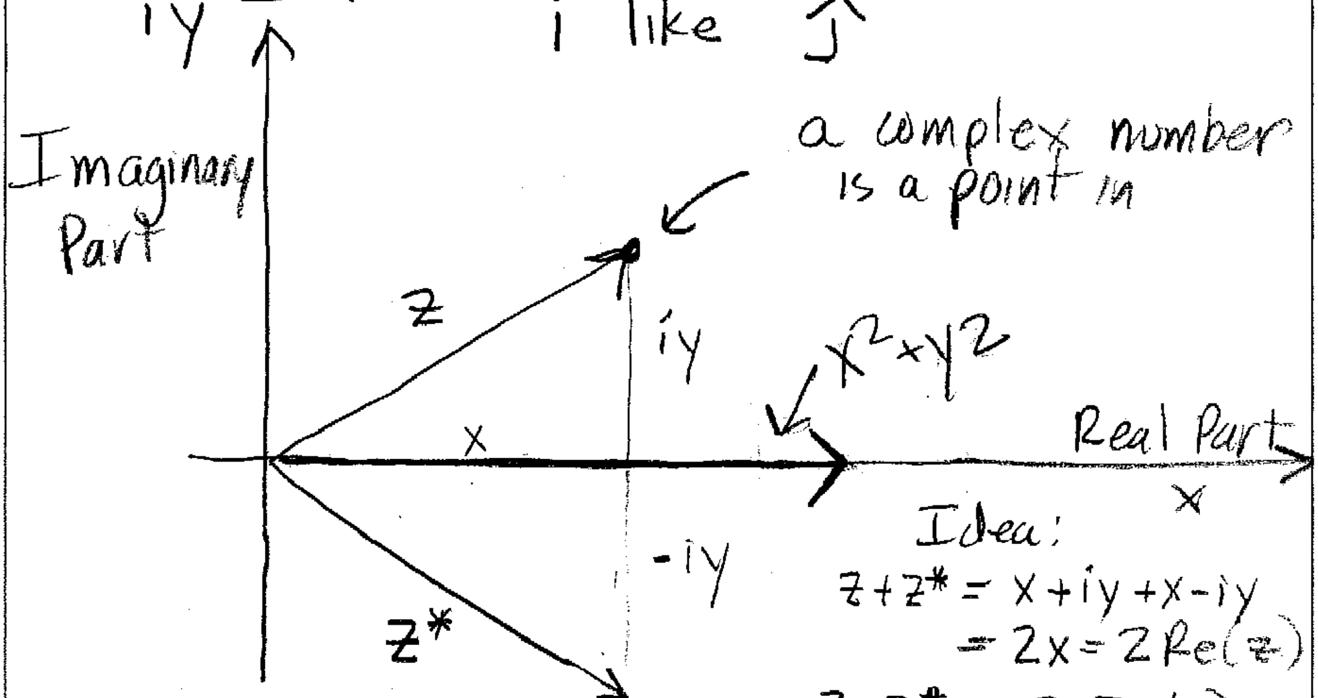


Idea:

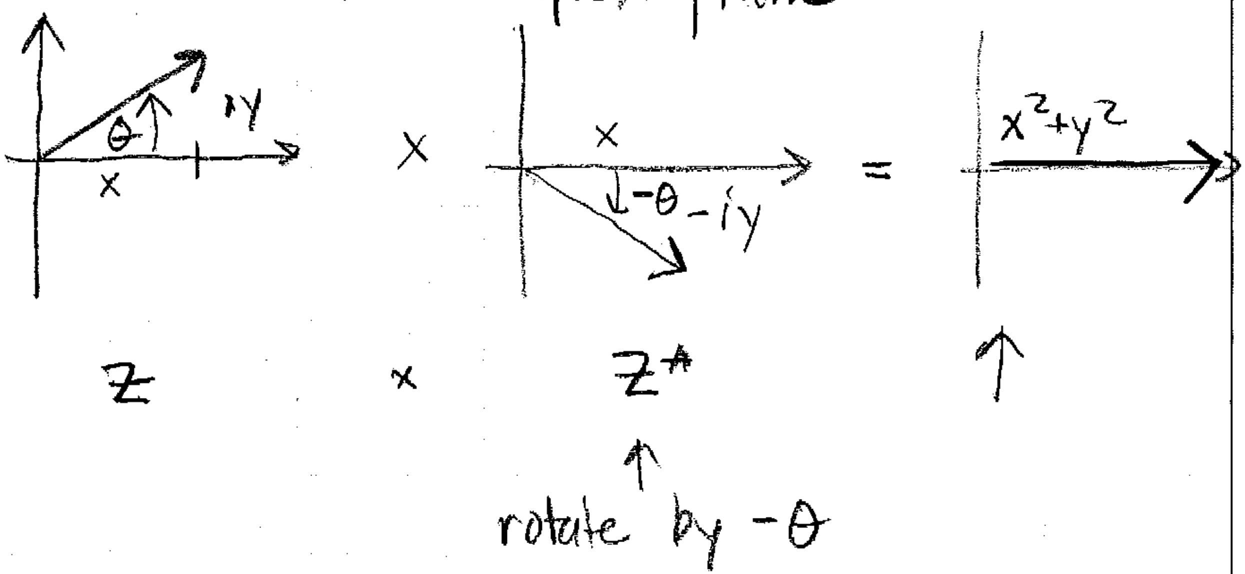
$$z+z^* = x+iy+x-iy = 2x = 2\,\mathrm{Re}(z)$$

$$z-z^* = 2i\,\mathrm{Im}(z)$$

$$z \cdot z^* = x^2+y^2$$

No matter what magnitude is, zz^* ends up on the Real Axis...

Idea: multiplication by a complex # constitutes a rotation in the complex plane



not a proof, but just an observation

Division

$$\frac{1}{i} \times i = 1 \qquad -i \cdot i = --1$$
$$= 1$$

so $\boxed{\frac{1}{i} = -i}$

Another view: $\frac{1}{i} \times \frac{i^*}{i^*} = \frac{i^*}{|i|^2}$

$$i \cdot i^* = i \cdot (-i) = -i^2 = --1 = 1$$

$$\frac{1}{i} = \frac{i^*}{1} = i^* = -i$$

$$\frac{1}{x+iy} = \frac{1}{x+iy} \cdot \frac{(x+iy)^*}{(x+iy)^*}$$

$$\boxed{\frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}, \quad \frac{1}{z} = \frac{z^*}{|z|^2}}$$

$$\boxed{e^{i\pi} + 1 = 0}$$

$$\boxed{e^{i\theta}}$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$$

$$\frac{d}{dx}(e^x) = 0 + 1 + \frac{\cancel{2}}{\cancel{2}}x + \frac{\cancel{3}}{\cancel{3}\cdot 2}x^2 + \frac{\cancel{4}}{\cancel{4}\cdot 3\cdot 2}x^3 + \dots$$
$$= e^x$$

$$e^{i\theta} = 1 + i\theta + \frac{1}{2}(i\theta)^2 + \frac{1}{3!}(i\theta)^3 + \frac{1}{4!}(i\theta)^4 + \ldots$$

$$= 1 \quad - \frac{1}{2}\theta^2 \quad + \frac{1}{4!}\theta^4 + \ldots$$

$$i \times \left[\theta \quad - \frac{1}{3!}\theta^3\right.$$

$i^3 = -i$ $\quad$ $i^4 = 1$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$e^{i\pi} = \cos\pi + i\sin\pi = -1$$

so $\boxed{e^{i\pi} + 1 = 0}$ $\quad (e^{i\theta})^* = e^{-i\theta}$

$$|e^{i\theta}|^2 = (\cos\theta + i\sin\theta)\underbrace{(\cos\theta - i\sin\theta)}_{(e^{i\theta})^*}$$

$$= \cos^2\theta + \sin^2\theta = 1$$

$e^{i\theta}$ is a type of unit vector...