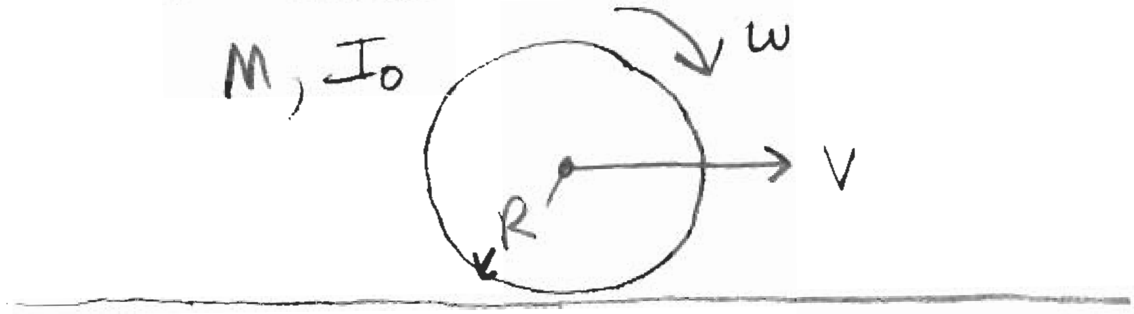
- rotation axis through center of mass



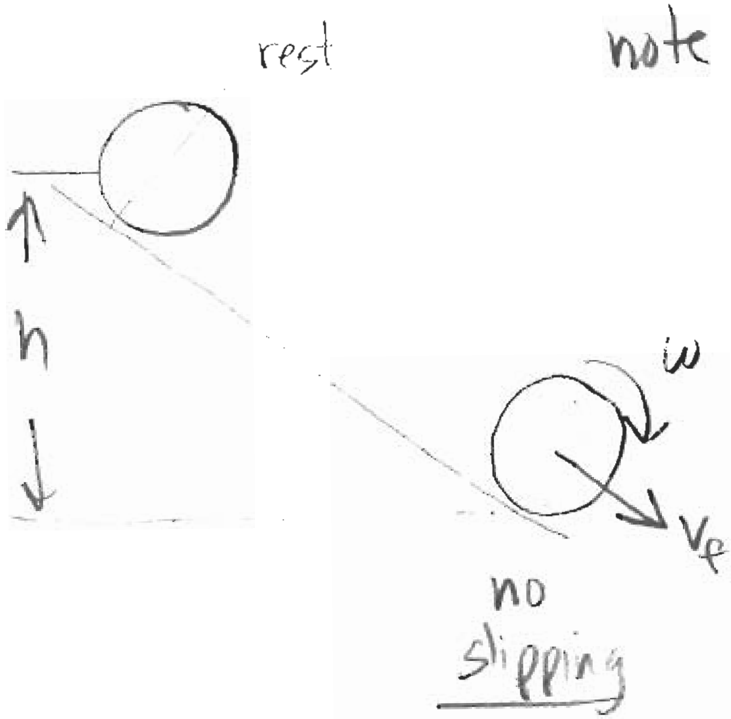
$v = \omega R$ (no slipping)

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I_0\omega^2$$

↑ $\omega = \frac{v}{R}$

$$K = \frac{1}{2}Mv^2\left(1 + \frac{I_0}{MR^2}\right)$$

↑ note, $\frac{I}{MR^2} < 1$



$$Mgh = \frac{1}{2}Mv_f^2\left(1 + \frac{I_0}{MR^2}\right)$$

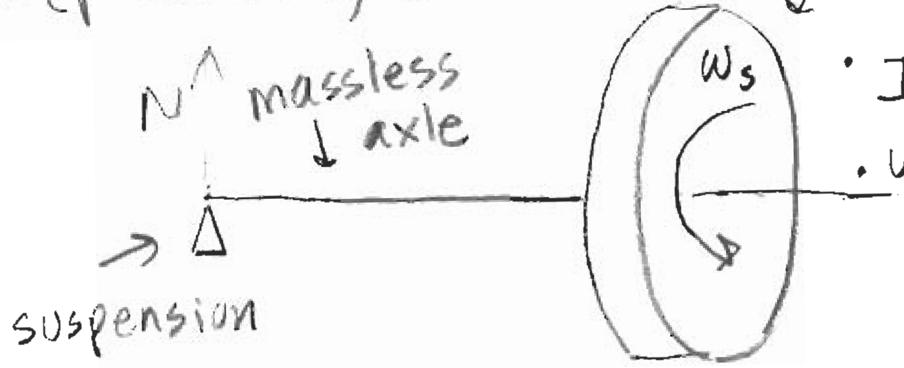
$$v_f = \sqrt{\frac{2gh}{1 + \frac{I_0}{MR^2}}}$$

ring largest $\frac{I_0}{MR^2} = 1$

smallest $v_f = \sqrt{gh}$ up to $\sqrt{2gh}$

The Gyroscope

Two angular velocities, ω (spin) + Ω (precession).



doesn't fall!

Net Force

↑ N (at suspension)

↓ −Mg (at mass)

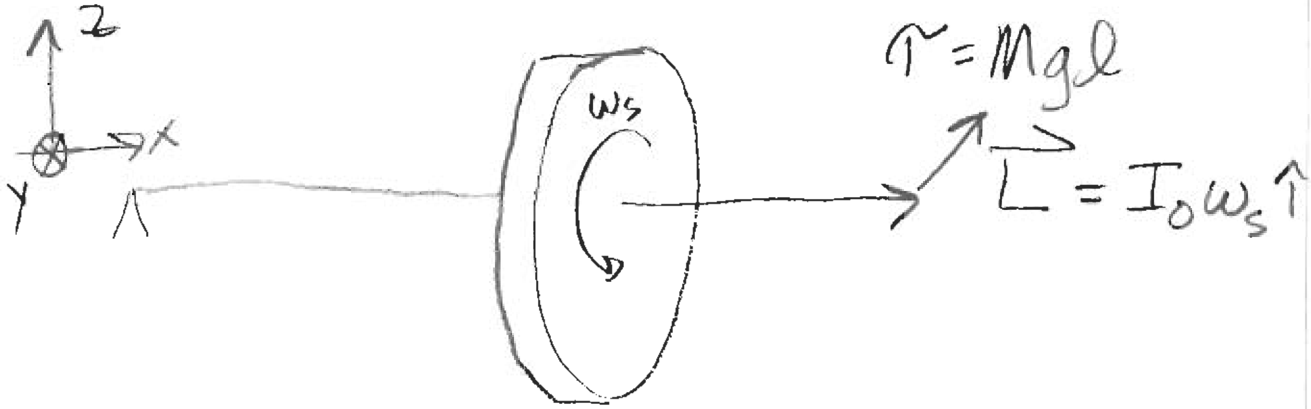
can cancel

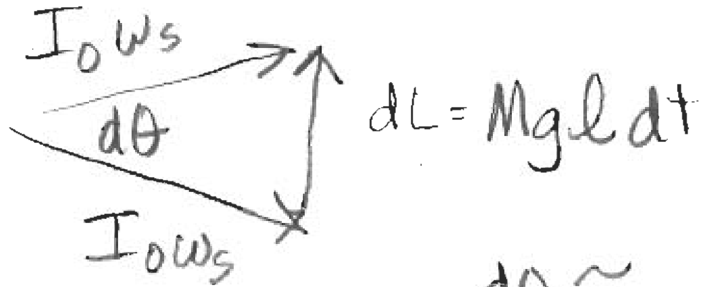
N = Mg

Torque?

$\tau = N\ell$
$= Mg\ell$

But, $\vec{\tau} = \frac{d\vec{L}}{dt}$ $\vec{L} \neq 0$ here!





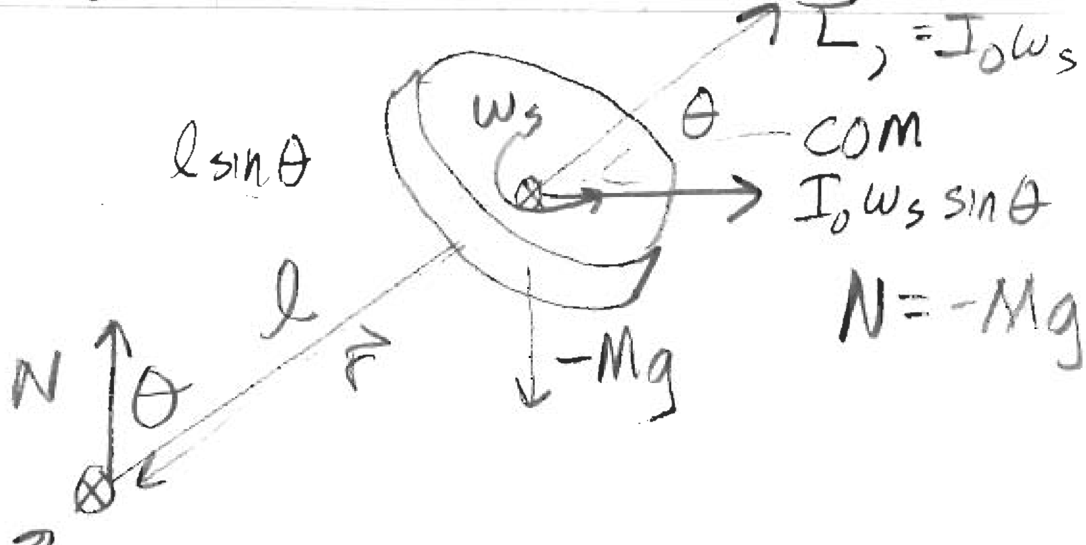
$$d\theta \simeq \frac{Mg\ell}{I_0 \omega_s} dt$$

$$\Omega \equiv \frac{d\theta}{dt} = \frac{Mg\ell}{I_0 \omega_s} \propto \frac{1}{\omega_s}$$

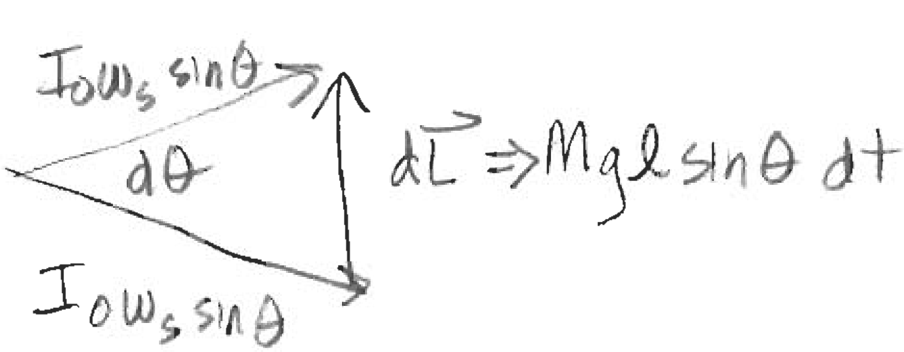
note: $\frac{M}{I_0} = \frac{M}{f \cdot MR^2} = \frac{1}{fR^2}$

- mass independent
- big I_0, small Ω

Make an angle between "spin" axis and vertical ...



$\vec{\tau} = Mgl\sin\theta$



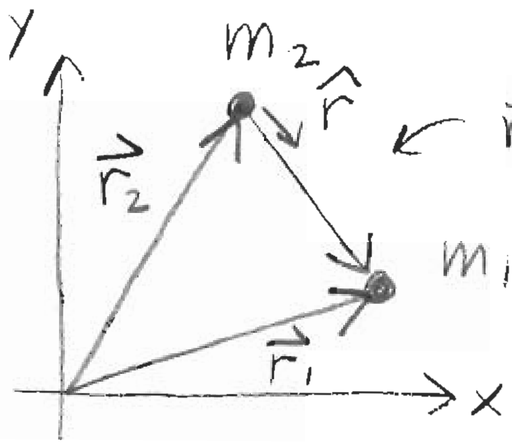
$$d\theta \simeq \frac{Mgl\sin\theta}{I_0\omega_s\sin\theta}\,dt$$

$$\boxed{\Omega = \frac{d\theta}{dt} = \frac{Mgl}{I_0\omega_s}}$$ same!

Chapter 9 – Central Force

2 bodies $\Rightarrow$ 1 body ☺

3 bodies $\not\Rightarrow$ 2 bodies (☹)



$\leftarrow \vec{r}_1 - \vec{r}_2 \equiv \vec{r}$

$r \equiv |\vec{r}|$

$\vec{F}$ of m_2 on m_1

$\equiv f(r)\,\hat{r}$

$f(r)$ depends on $r = |\vec{r}|$

> 0 : m_1 repelled

< 0 : m_1 attracted

so $\quad m_1 \ddot{\vec{r}}_1 = f(r)\,\hat{r}$ depends on $\vec{r}_2$

Newton's Third $\quad m_2 \ddot{\vec{r}}_2 = -f(r)\,\hat{r}$ depends on $\vec{r}_1$

"coupled" equations

Change variables.

$$\left.\begin{matrix}\vec{r}_1 \\ \vec{r}_2\end{matrix}\right\} \Rightarrow \begin{matrix}\vec{r} \equiv \vec{r}_1 - \vec{r}_2 \\ \vec{R} \equiv \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}\end{matrix}$$

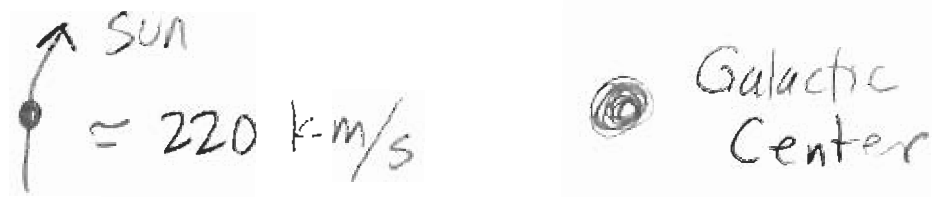
$\ddot{\vec{R}} = 0$ (no external forces) or $m_1\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}_2 = f(r)\hat{r} - f(r)\hat{r} = 0$

(Newton 3) $= \ddot{\vec{R}}$

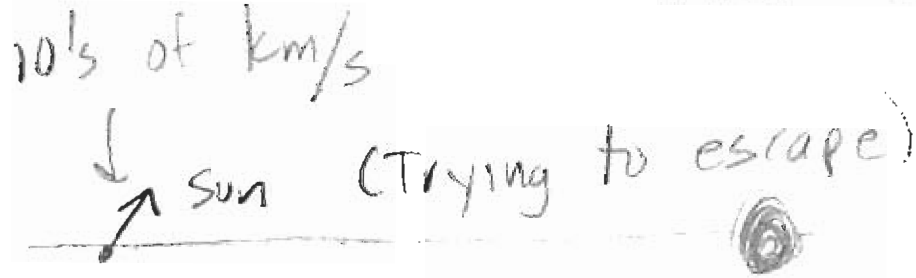
$\vec{R} = \vec{R}_0 + \vec{V}t$ } pick coordinates, "boost" reference frame to move.

Sun is moving.

From top



From Side



But we work problems assuming sun at rest, and we usually put sun at origin

$\ddot{\vec{r}}_1 = \frac{1}{m_1} f(r)\hat{r}$

$\ddot{\vec{r}}_2 = -\frac{1}{m_2} f(r)\hat{r}$

$\ddot{\vec{r}} = \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) f(r)\hat{r}$

$\frac{1}{\;} = \frac{1}{\;} + \frac{1}{\;}$