Decomposition

$\boxed{\text{Is } \alpha = 0 \text{ ?}}$

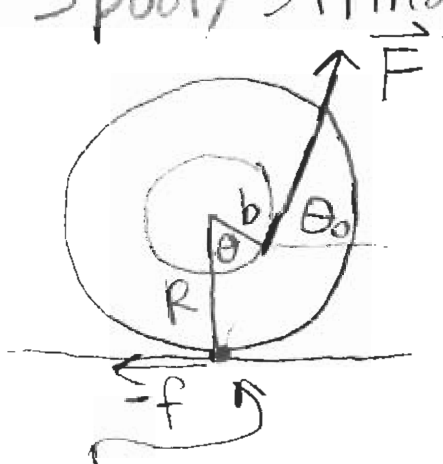
Yes "Statics"

Can pick **any** point to evaluate torques about

↓

example

Spool/String



pick this point

→ torque of $f = 0$

→ torque of $\vec{F} = 0$ (when $\vec{F}$ aimed at contact point)

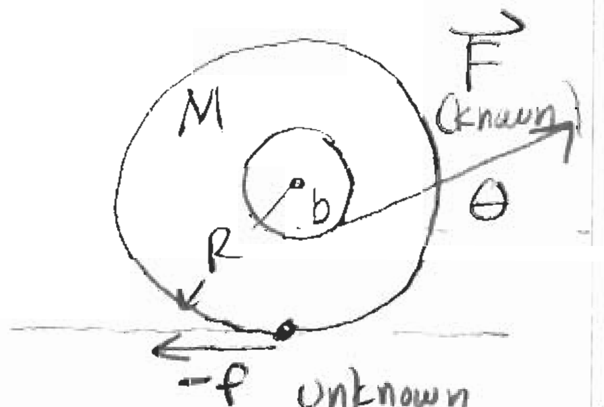
$$\cos\theta_0 = \frac{b}{R}$$

No

(1) Net Force acts on the center of mass

(2) Compute Torques about the center of mass

(3) $\vec{L}_{tot} = \vec{L}_{\text{of cm}} + \vec{L}_{\text{about cm}}$



2 unknowns

Forces

horizontal: $F\cos\theta - f = Ma_x$

vertical: $F\sin\theta + N - Mg = 0$ (unk)

$$N = Mg - F\sin\theta$$

Torque

$\alpha \neq 0$, use center of mass...

$$Fb - fR = -I\alpha$$

$\alpha R = a_x$

note F makes $-\alpha$

$$F\cos\theta - f = M\alpha R$$

$$F\frac{b}{R} - f = -\frac{I}{R}\alpha$$

$$F\left(\cos\theta - \frac{b}{R}\right) = \left(MR + \frac{I}{R}\right)\alpha$$

$$\boxed{\alpha = \frac{\cos\theta - \frac{b}{R}}{MR + \frac{I}{R}} F}$$

$= 0$ when $\cos\theta = \frac{b}{R}$

$$F\cos\theta - f = MR \cdot \left(\frac{\cos\theta - \frac{b}{R}}{MR + \frac{I}{R}}\right) F$$

$$F\cos\theta - f = \frac{\cos\theta - \frac{b}{R}}{1 + \frac{I}{MR^2}} F$$

$$f = F\left(\cos\theta - \frac{\cos\theta - \frac{b}{R}}{1 + \frac{I}{MR^2}}\right)$$

$$= F\left(\frac{\cos\theta + \frac{I}{MR^2}\cos\theta - \cos\theta + \frac{b}{R}}{1 + \frac{I}{MR^2}}\right)$$

$$\boxed{f = \left(\frac{\frac{b}{R} + \frac{I}{MR^2}\cos\theta}{1 + \frac{I}{MR^2}} \right) F}$$

note $f > 0$ always,

note: when $\cos\theta = \frac{b}{R}$, $f = \frac{b}{R} F$ $\overleftarrow{-f}$

$f = 0$ when $\cos\theta_f = \frac{b}{R} \times \frac{MR^2}{I}$

doesn't always happen.

Starts to slide when

$$|f| = \mu N \quad \text{or}$$

$$\left| \frac{\frac{b}{R} + \frac{I}{MR^2}\cos\theta}{1 + \frac{I}{MR^2}} \right| F = \mu \cdot (Mg - F\sin\theta)$$

Energy

$\tau = I\alpha \Rightarrow$ rotational energy

is $\frac{1}{2} I \omega^2$ ... when rotation axis is stationary

When ...
- rotation axis is moving
- rotation axis always pointed in the same direction