

Stick (about center)



center of mass → center ...  $I_0 = \frac{1}{12} ML^2$

Stick (about end)



axis

$$I = \frac{1}{3} ML^2$$

example of the parallel axis theorem

$$I = I_0 + M l^2$$

↑  
moment  
of inertia  
about center  
of mass

↑  
l is distance  
from center  
of mass  
to new axis

$$I = \frac{1}{12} ML^2 + M \left( \frac{L}{2} \right)^2$$

$$= \left( \frac{1}{12} + \frac{1}{3} \right) ML^2 = \left( \frac{1+4}{12} \right) ML^2$$

$$I = \frac{1}{3} ML^2$$

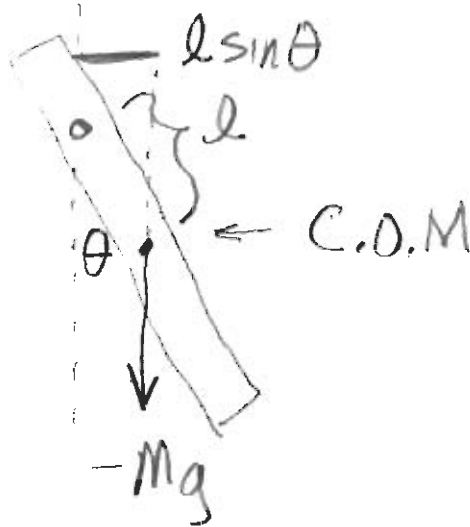
$$F = ma \longrightarrow \tau = I \alpha$$

$$\alpha = \dot{\omega} = \ddot{\theta}$$

↑  
ω of rotation!

## Physical Pendulum

Extended Object made into pendulum  
Like a... stick!



torque... into page

$$= -Mg l \sin \theta$$

$$= I \ddot{\theta}$$

$$I = I_0 + M l^2$$

↑  
about  
center of mass =  $\frac{1}{12} M L^2$

$$\left( \frac{1}{12} M L^2 + M l^2 \right) \ddot{\theta} = -Mg l \theta$$

$$\ddot{\theta} = -\omega^2 \theta \quad ; \quad \omega^2 = \frac{g l}{\frac{1}{12} L^2 + l^2}$$

ω of SHO

$$\rightarrow \frac{g}{l} \quad (l \gg L)$$

$$\rightarrow \frac{12}{L} g \frac{l}{12} \quad (l \ll L)$$

# Rolling (no slipping)



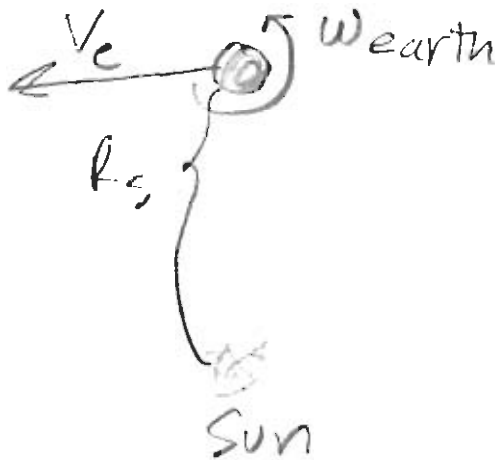
$\rightarrow$  that point at rest

$$v - \omega R = 0$$

$$v = \omega R$$

$$\dot{v} = a = \dot{\omega} R = \alpha R$$

$$L_{\text{tot}} = L_{\text{COM}} + L_{\text{Internal Rotation}}$$



(Fixed axis)

$$L_{\text{tot}} = M_e v_e r_s + I_e \omega_e$$