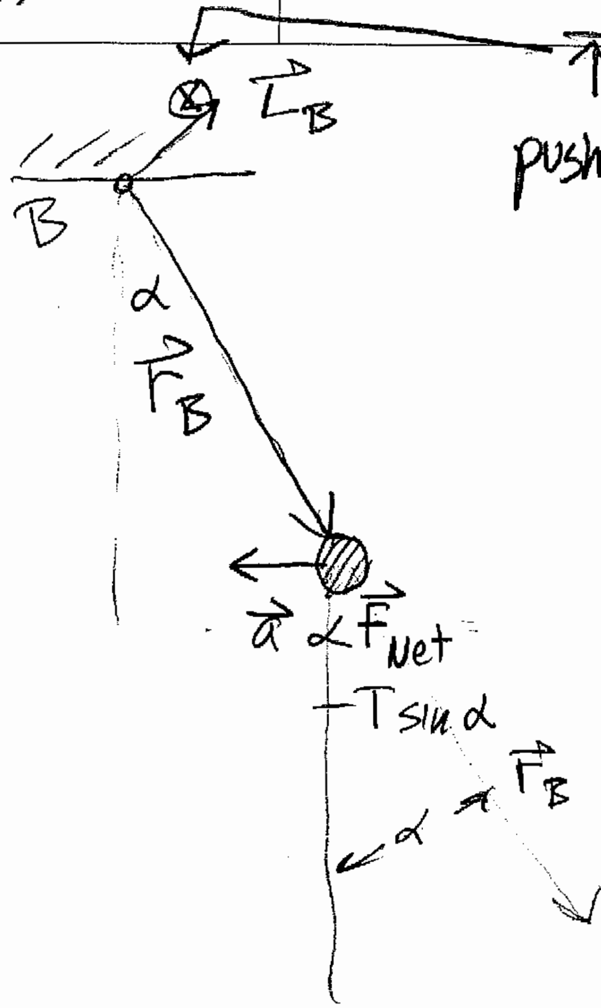
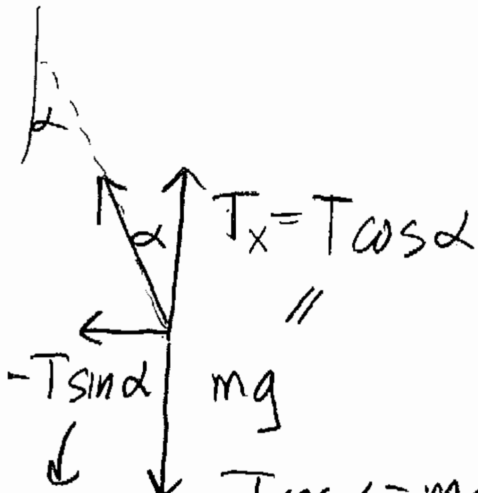
$\vec{\tau}_B$ pushes $\vec{L}_B$ around.





$T\cos\alpha = mg$

$T\sin\alpha = mr\omega^2$

$\vec{\tau}_B = \vec{r}_B \times \vec{F}_{Net}$ direction in the page.

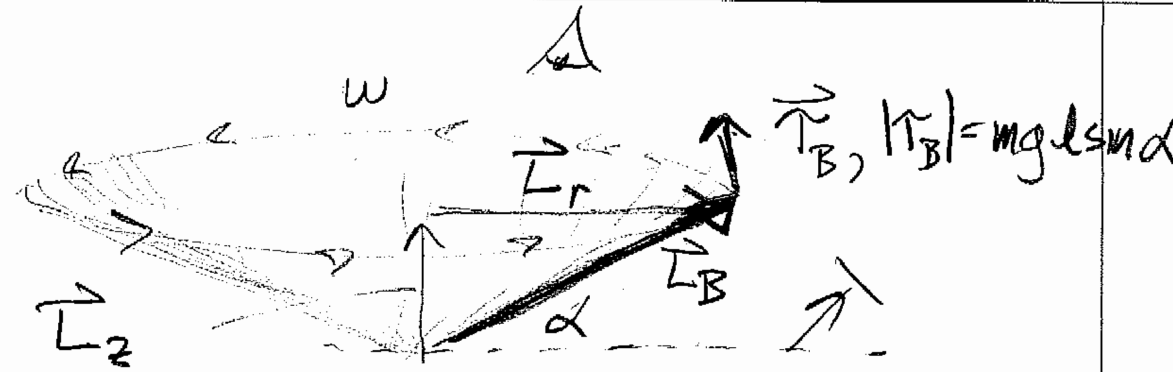
<u>magnitude</u>: $|\vec{r}_B||\vec{F}_{Net}|$

$$\ell \cdot T\sin\alpha \cdot \underbrace{\sin(90+\alpha)}_{\cos(-\alpha)}$$

$$|\vec{\tau}_B| = \ell \cdot \underbrace{T\cos\alpha}_{mg} \cdot \sin\alpha$$

$$\boxed{|\vec{\tau}_B| = mg\ell\sin\alpha}$$

$$\frac{d\vec{L}_B}{dt} = \vec{\tau}_B \neq 0$$



$|\vec{L}_B| = lr\omega$

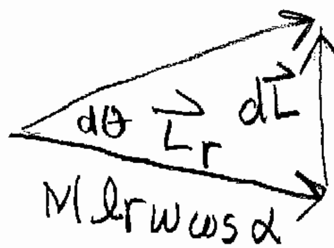
$|\vec{L}_z| = ml r\omega \sin\alpha$

$l\sin\alpha = r$

$|\vec{L}_z| = \underbrace{mr^2}\omega$

$|\vec{L}_r| = mlr\omega\cos\alpha$

$d\vec{L} = \vec{\tau}_B dt$



$|\vec{\tau}_B|\, dt = d\vec{L}$??

check

$d\theta = \omega dt$

$mgl\sin\alpha\, dt$

$|\vec{L}_r|\omega dt$

$= mlr\omega\cos\alpha\,\omega dt$

$= \underbrace{mr\omega^2}\, l\cos\alpha\, dt$

$= T\sin\alpha\, l\cos\alpha\, dt$

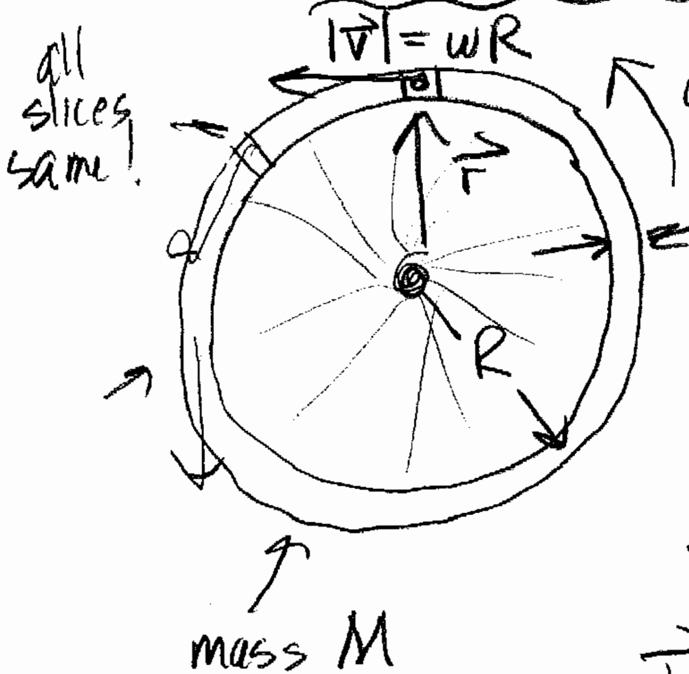
$= \underbrace{T\cos\alpha}\, l\sin\alpha\, dt$

$\boxed{mgl\sin\alpha\, dt = mgl\sin\alpha\, dt}$

Yes, agrees.

Fixed Axis Rotation

Bicycle Wheel



what is $\vec{L}$?

$$\vec{L} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times M\vec{v}$$
$$\vec{L} = MR \cdot \omega R \; \hat{k}$$
$$\vec{L} = \underbrace{(MR^2)}_{\text{Think } M} \underbrace{\omega}_{V} \; \hat{k}$$

$$\vec{L} = I \omega \hat{k}$$

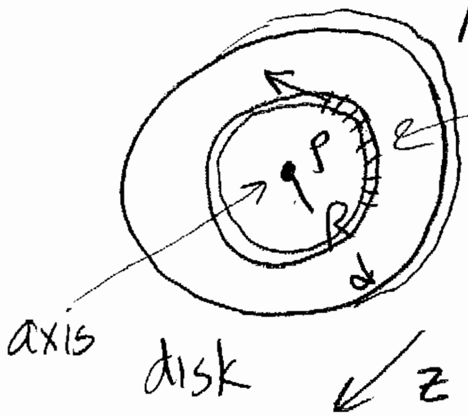
"moment of inertia" like mass

Ring: MR^2

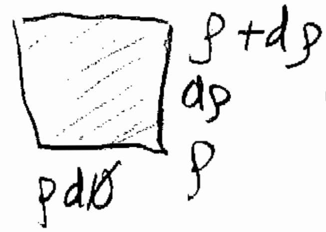
Other shapes: $\neq MR^2$!

Fixed Axis Rotation

$\approx$ 2-d object



all of these are at radius ρ



$\leftarrow$ area is $d\rho\,(\rho\, d\phi)$

$$dA = \rho\, d\rho\, d\phi$$

<u>mass</u>? How much mass per unit area?

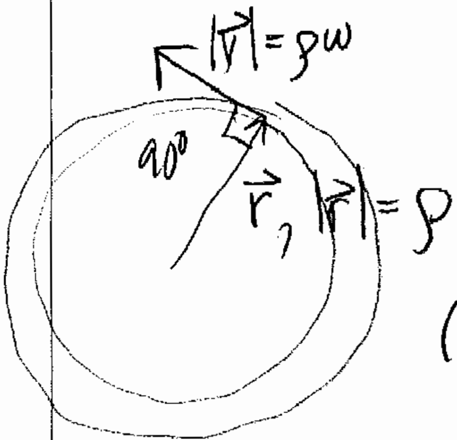
assume uniform thickness

$$\sigma = \frac{M}{\pi R^2} \leftarrow \frac{\text{mass}}{\text{area}}$$

$$dm = \sigma\, dA = \frac{M}{\pi R^2} \rho\, d\rho\, d\phi$$

How much angular momentum?

$$d\vec{L} = \vec{r} \times (dm\, \vec{v}) = \rho\, dm\, (\rho\omega)\, \hat{k}$$



$$d\vec{L} = dm\, \rho^2 \omega\, \hat{k}$$

now do the whole ring at same ρ

$$(d\vec{L})_{ring} = \rho^2 \omega \int_0^{2\pi} \frac{M}{\pi R^2} \rho\, d\rho\, d\phi\, \hat{k}$$

$$(d\vec{L})_{ring} = \rho^2 \omega \quad 2\cancel{\pi} \frac{M}{\cancel{\pi} R^2} \rho \, d\rho$$

$$= \frac{2M}{R^2} \omega \ \rho^3 d\rho \ \hat{k} \quad \left(\begin{matrix}\text{Ring as} \\ \text{part}\end{matrix}\right)$$

$$\vec{L}_{disk} = \frac{2M}{R^2} \omega \hat{k} \left(\int_0^R \rho^3 d\rho = \frac{1}{4} \rho^4 \Big|_0^R \right)$$

$$= \frac{2M}{R^2} \omega \hat{k} \ \frac{1}{4} R^4 = \left(\frac{1}{2} M R^2\right) \omega \hat{k}$$

$$\vec{L}_{disk} = I_{disk} \ \omega \hat{k}$$

$$\hookrightarrow I_{disk} = \frac{1}{2} M R^2$$