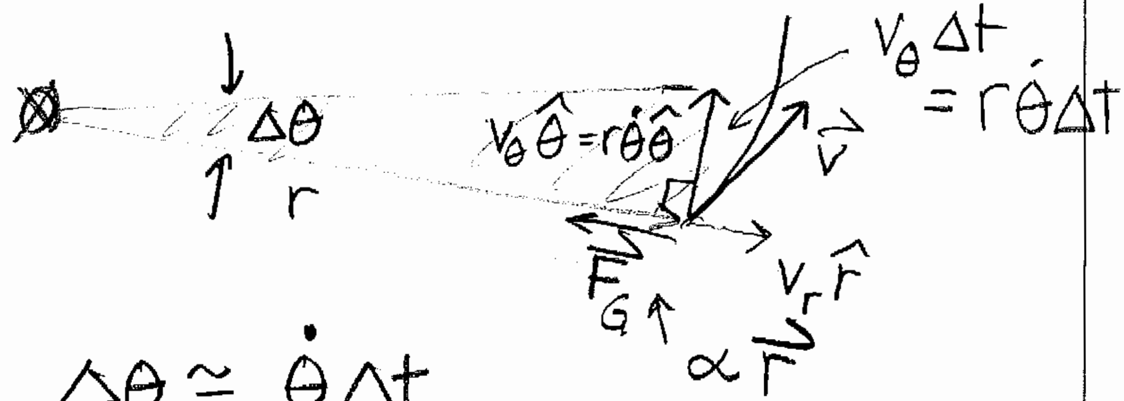
Let Δt be very small...



$$\Delta\theta \simeq \dot{\theta}\Delta t$$

$$\Delta A = \frac{1}{2} r \cdot r\dot{\theta}\Delta t = \frac{1}{2} r \cdot v_\theta \Delta t$$

$$= \frac{1}{2}\Delta t \cdot \frac{1}{m}\underbrace{(r m v_\theta)}_{L_z}$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2}\frac{L_z}{m} = \text{constant.}$$

if $$\frac{dA}{dt} = \text{constant} = \frac{1}{2}\frac{L_z}{m}$$

$$\boxed{A = \frac{1}{2}\frac{L_z}{m} \cdot t}$$

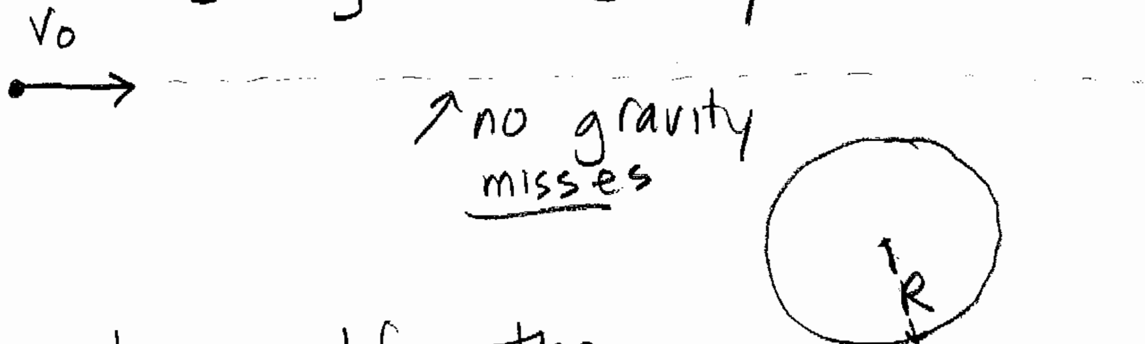
$$\Delta A \propto \Delta t$$

P21: Final 58 ± 23 Overall 71 ± 18

Capture Cross Section of a Planet

important concept.

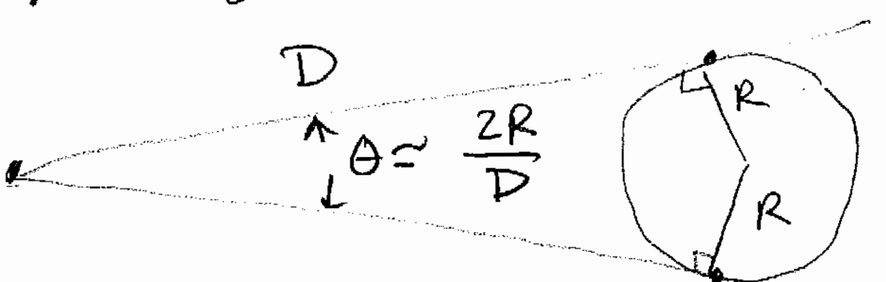
① imagine no gravity



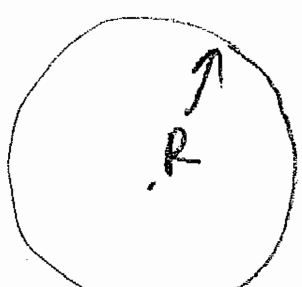
How to quantify the tendency to hit or miss?

Bigger planets (R bigger) have a bigger tendency...

One way: angle (not cross section)



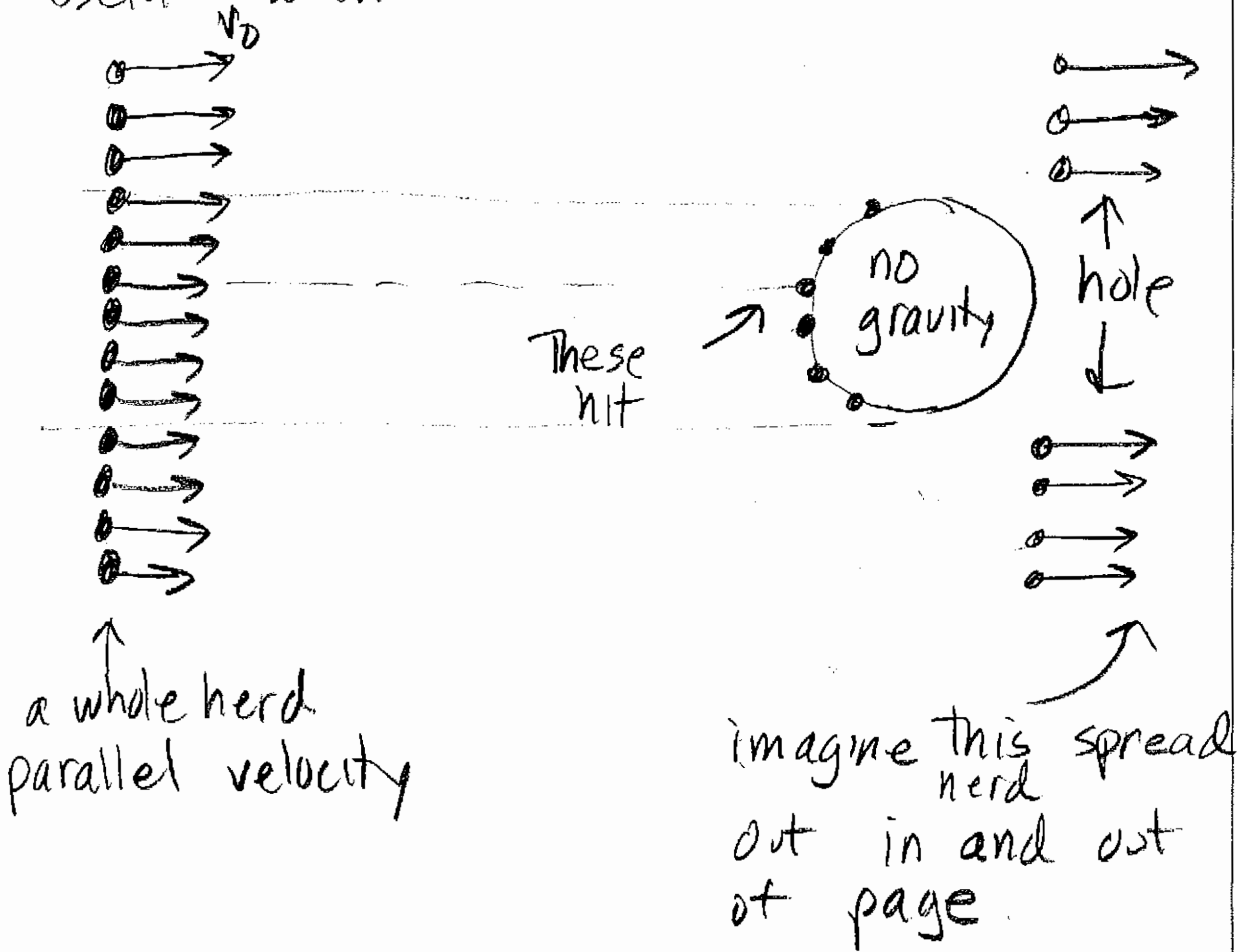
Another way: Area of the planet.

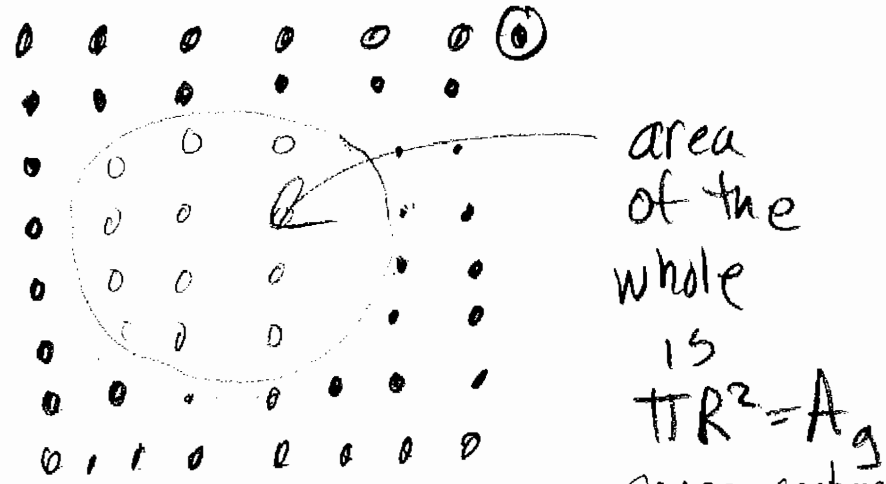


$A_g = \pi R^2$

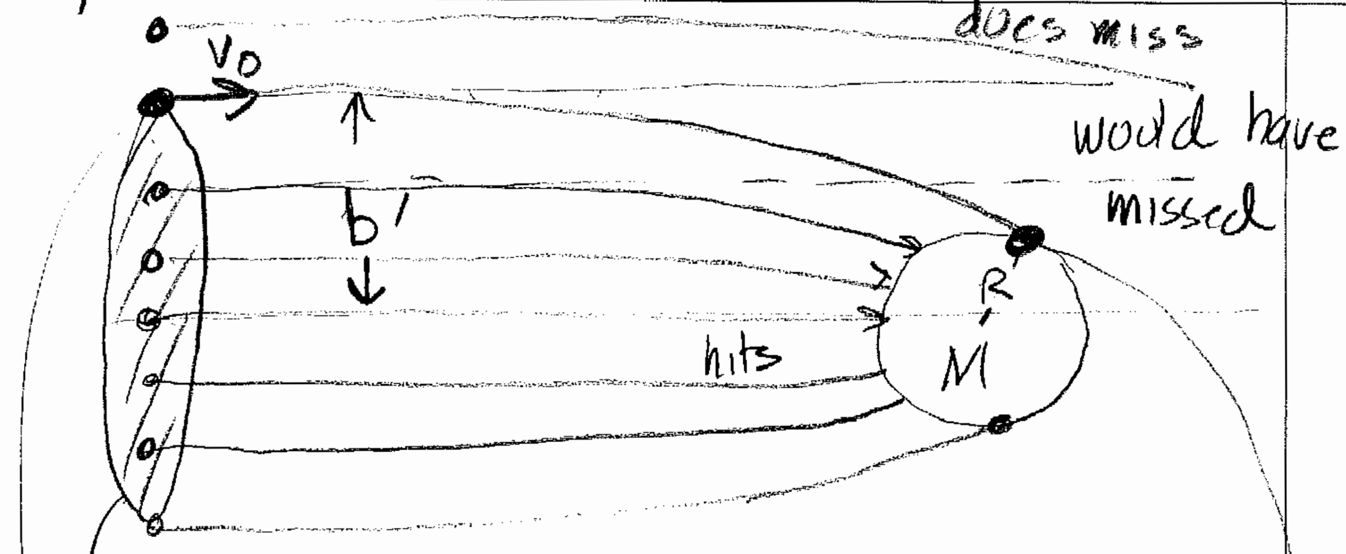
= "cross section" with zero gravity

Usefull when:





Now.. turn on gravity...
replay this situation...



$$A_e = \pi b'^2 > A_g = \pi R^2$$

How can we evaluate b'?

→ Angular Momentum + Energy

∞ far away: $\quad E_i = \frac{1}{2} m v_0^2$

$$E_f = \frac{1}{2} m v^2 - G\frac{Mm}{R}$$

$$E_i = E_f$$

$$\frac{1}{2}\cancel{m} v_0^2 = \frac{1}{2}\cancel{m} v^2 - G\frac{M\cancel{m}}{R}$$

$$v^2 = v_0^2 + \frac{2GM}{R}$$

$\underline{L}$: ∞ far away w/r to planet center...

$$= m v_0 b'$$ ← "moment arm"

finally … $mvR = mv_0 b'$

$$b' = \frac{v}{v_0} R \quad (>R)$$

$$\pi b'^2 = \pi R^2 \times \frac{v^2}{v_0^2}$$

$$= \pi R^2 \cdot \left(\frac{v_0^2 + \frac{2GM}{R}}{v_0^2} \right)$$

$$\boxed{A_e = \pi b'^2 = \pi R^2 \left(1 + \frac{2GM}{R v_0^2}\right)}$$

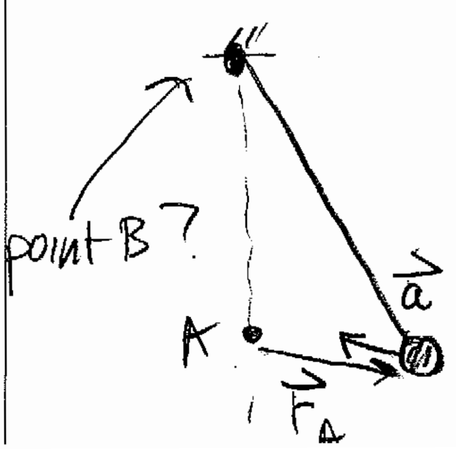
① $G \to 0$, no gravity $= \pi R^2 = A_g$

② $v_0 \to 0$, $\to \infty$!

③ $R \to 0$ $\quad A_e = \pi R^2 + \underbrace{\frac{2\pi R GM}{v_0^2}}$

$R \to$ very small, this term dominates.

Torque on the Conical Pendulum



$$\vec{a} = \frac{\vec{F}_{Net}}{m} \propto -\vec{r}_A$$

$$\vec{\tau}_A = \vec{r}_A \times \vec{F}_{Net} \propto \vec{r}_A \times (-\vec{r}_A)$$

$$\propto 0$$

$\frac{d\vec{L}_A}{dt} = 0$ therefore … consistent.