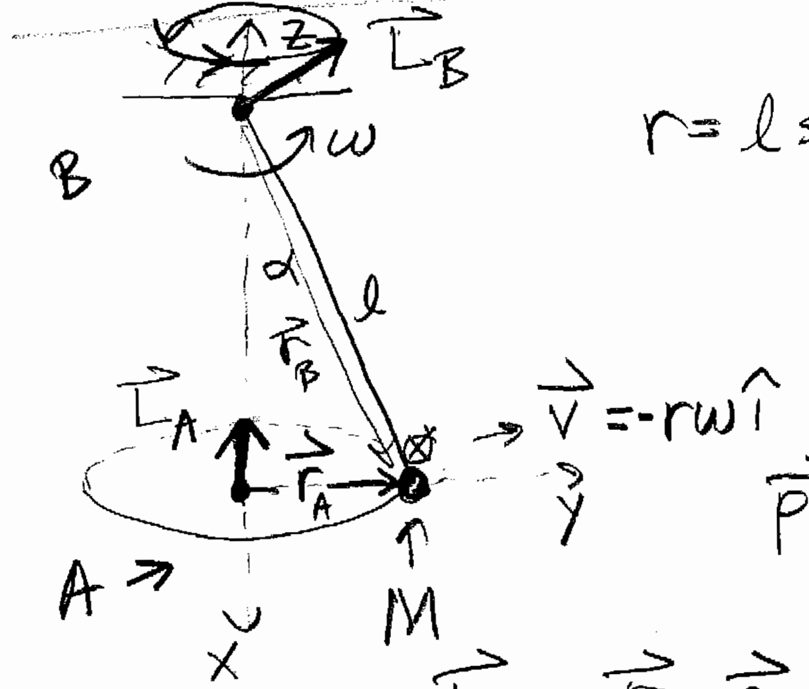
Conical Pendulum



$r = l \sin\alpha$

$\vec{p} = -Mr\omega\hat{\imath}$ is $\perp$ to $\vec{r}_A$

$\vec{L}_A = \vec{r}_A \times \vec{p} = \underbrace{Mr^2}\omega\hat{k}$

note this "moment of inertia"

Constant in magnitude + direction.

$\vec{L}_B = \vec{r}_B \times \vec{p}$ $\quad$ $\vec{r}_B$ + $\vec{p}$ also $\perp$!

$|\vec{L}_B| = |\vec{r}_\perp||\vec{p}| = lMr\omega = Mrl\omega$

<u>magnitude</u> never changes

<u>direction</u> always in the same plane as mass + suspension point, $\perp$ to rope.

<u>Note</u>: z-component of $\vec{L}_B$

$= Mrl\omega \sin\alpha = Mr^2\omega$. same as $\vec{L}$.

Torque

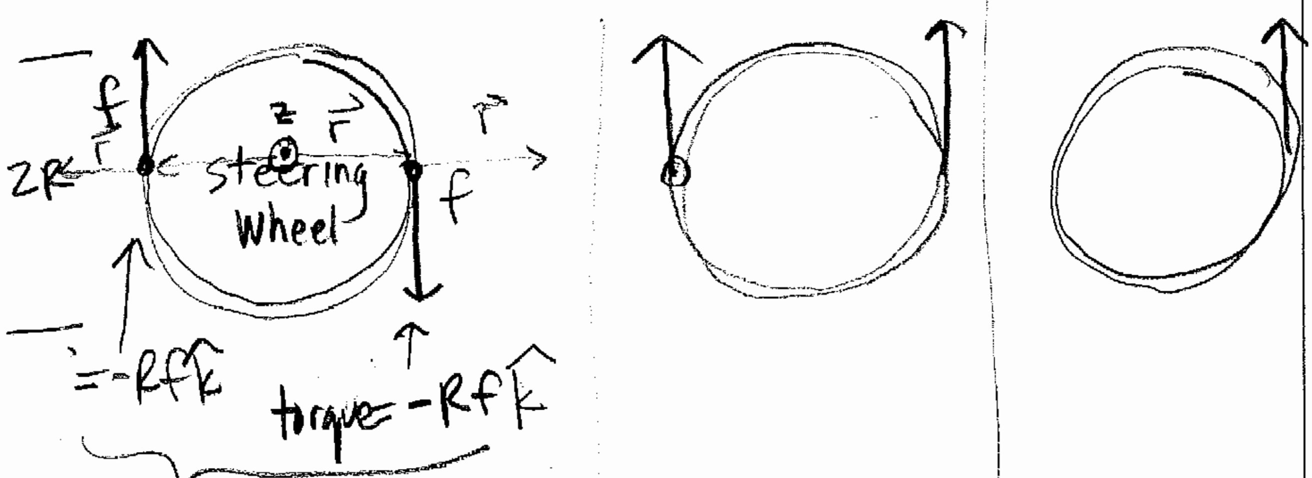
$\vec{\tau} = \vec{r} \times \vec{F}$ ... can decompose just like $\vec{r} \times \vec{p}$

$|\vec{\tau}| \propto |\vec{r}|$ ... the bigger $|\vec{r}|$, the more torque

$\propto$ sin(angle between)

push $\perp$!

Not the same as force.



add up...

Net Torque ...	$-2fR\hat{k}$	$= 0$	$= +fR\hat{k}$
Net Force ...	0	$= 2f\hat{j}$	$= f\hat{j}$

Torque $\Leftrightarrow$ Angular Momentum

$\frac{d}{dt}(\vec{L}) = \frac{d}{dt}(\vec{r} \times \vec{p})$ $\Leftarrow$ chain rule works!

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \underbrace{\frac{d\vec{p}}{dt}}$$

$$= \underbrace{\vec{v} \times m\vec{v}}_{0} + \vec{r} \times \vec{F}_{Net} \quad \Big\} \; \vec{F}_{Net} = \frac{d\vec{p}}{dt}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}_{Net}}$$

When $\vec{F}_{Net} \propto \vec{r}$, $\frac{d\vec{L}}{dt} = 0$!

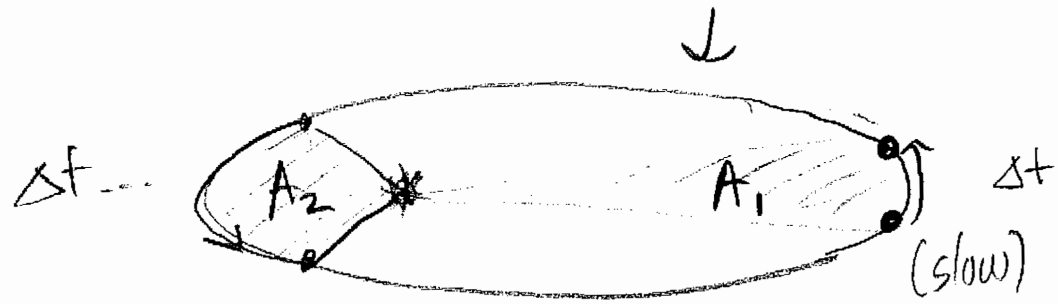
$\vec{L}$ = constant (ponder conical pendulum)

Most Famous Case... planets about sun...

Leads to Kepler's 2^{ND} Law...

In equal times, planet's orbits sweep out equal area

↓



In same Δt $A_1 = A_2$