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Beyond Rectilinear Systems

$x \rightarrow \Theta$ (when circular motion).

$\dot{x} \rightarrow \dot{\Theta} = \omega$

$\vec{p} = m\vec{v} \rightarrow$ angular momentum $\quad \underbrace{\vec{L} = \vec{r} \times \vec{p}}_{\text{for a particle.}}$

linear momentum

$\vec{F} \rightarrow$ torque $\quad \tau = \vec{r} \times \vec{F}$

force

mass $m \rightarrow$ Moment of Inertia $\quad I$

Fixed axis: $L = I\omega$

Concepts important when

1. Central Force... gravity, like planets orbiting sun
2. Solid, Rigid Bodies...
   a) Motion of center of mass is like that of a point particle with mass = mass of body
   b) Rotation about the center of mass (Chasle's Theorem)

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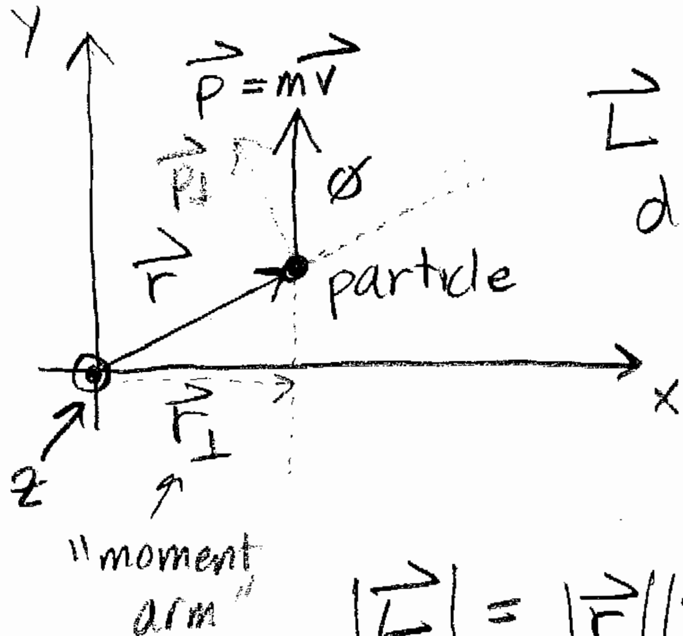
$\vec{L}$ for a point particle



$$\vec{L} \equiv \vec{r} \times \vec{p}$$

direction: Right Hand Rule

$\hat{k}$ here (matters little at the moment)

$$|\vec{L}| = |\vec{r}||\vec{p}|\sin\phi$$

$$= |\vec{r}|\,|\vec{p}_\perp| \qquad\qquad = |\vec{p}|\,|\vec{r}_\perp|$$

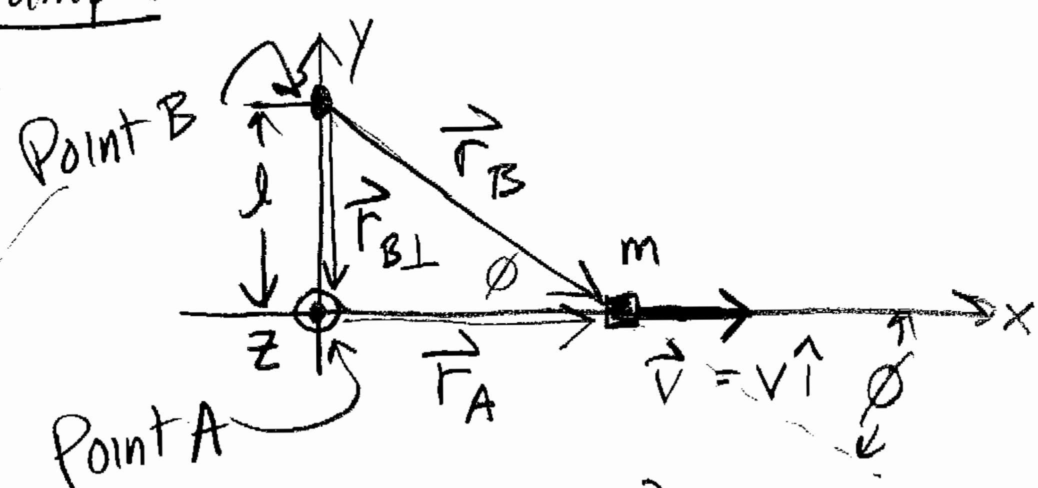
$|\vec{p}_\perp| = |\vec{p}|\sin\phi$ $\qquad$ $|\vec{r}_\perp| = |\vec{r}|\sin\phi$

also.. $\vec{r}\times\vec{p} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$ $\quad z \to 0,\ p_z \to 0$ when particle in a plane.

$$\boxed{\vec{r}\times\vec{p} = \hat{k}\begin{vmatrix} x & y \\ p_x & p_y \end{vmatrix}}$$

Note: the value of $\vec{L}$ depends on the origin of the coordinate system ($\vec{p}$ is not like this)!

Example 6.1



$$\vec{L}_A = \vec{r}_A \times (m\vec{v}) = 0$$

$$\vec{L}_B = \vec{r}_B \times (m\vec{v}) = (\quad) \times \hat{k}$$

$$|\vec{r}_B||m\vec{v}|\sin\phi$$

$$\underbrace{(|\vec{r}_B|\sin\phi)}_{|\vec{r}_{B\perp}| = \ell} mv$$

$$\vec{L}_B = mv\ell\hat{k}$$

another way... $\vec{r}_B = x\hat{\imath} - \ell\hat{\jmath}$

position of particle

$$\vec{r}_B \times \vec{p} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & -\ell & 0 \\ mv & 0 & 0 \end{vmatrix} = \hat{k}\begin{vmatrix} x & -\ell \\ mv & 0 \end{vmatrix}$$

$$\boxed{\vec{r}_B \times \vec{p} = mv\ell\hat{k} = \vec{L}_B} \neq \vec{L}_A$$