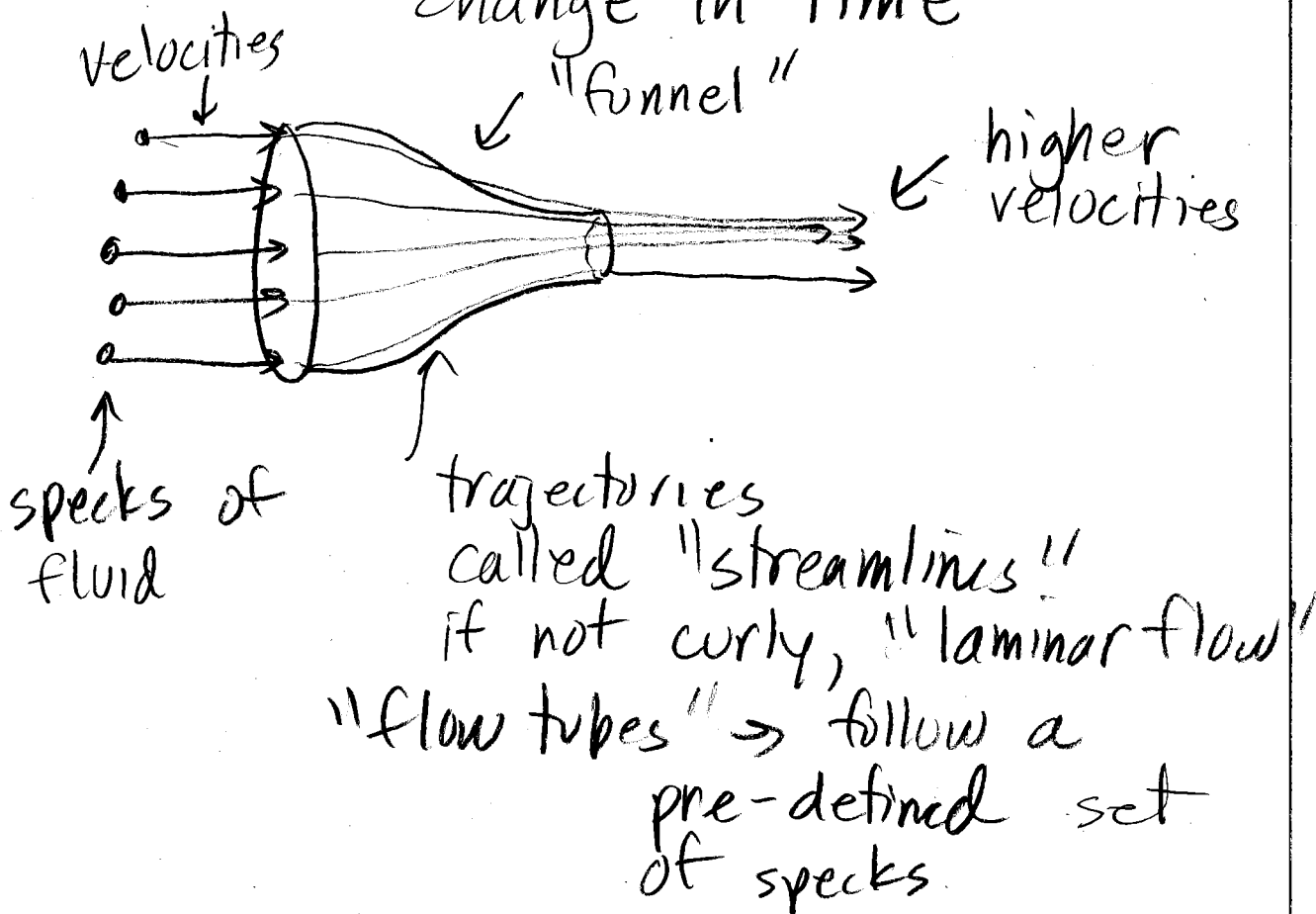


# Fluid Flow

Steady : pattern does not change in time



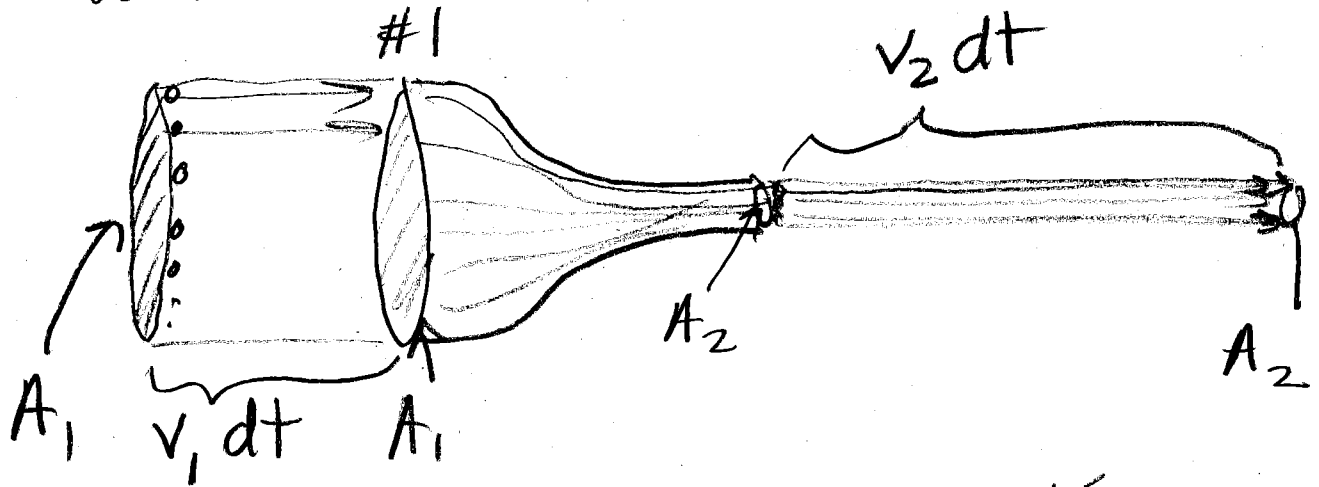
Whole thing above is a flow tube.

## Continuity Equation

Like "Play-Do" what goes in must come out

really, # atoms is what  $\propto$  mass

In time  $dt$ , compare volume in the flow tubes...



mass :  $\rho_1 A_1 v_1 dt = \rho_2 A_2 v_2 dt$

$$(\rho_1 v_1) \times A_1 = (\rho_2 v_2) \times A_2$$

"continuity equation"

• true when  $\vec{v} \perp$  to  $A$  rear

•  $\rho \vec{v} \equiv \vec{\phi} =$  "flux"

•  $\frac{dV}{dt} = v \cdot A =$  volume change w/r to time.

•  $\rho_1 = \rho_2$  for incompressible fluids

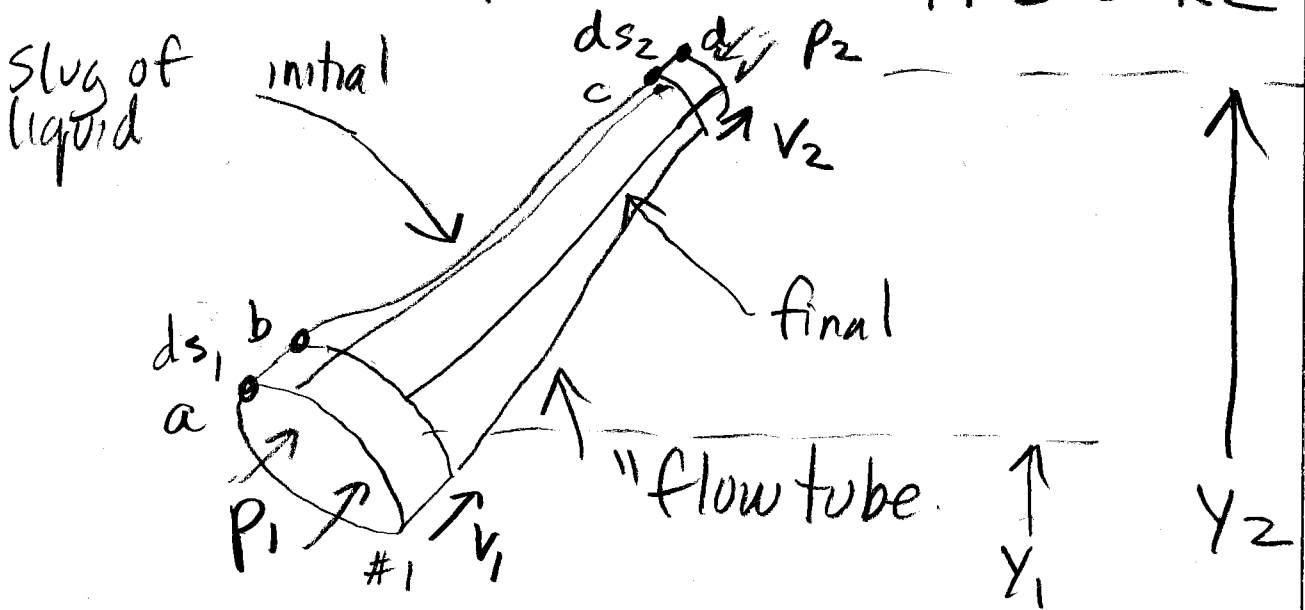
"Kinematics"

# Bernoulli's Principal

→ apply energy considerations to some fluid.

→ surprise result:

**FASTER = LESS PRESSURE**



$$dW = P_1 A_1 ds_1 - P_2 A_2 ds_2$$

$$= dU + dK$$

↑  
potential energy

↑ kinetic energy

assume incompressible

$$dU = - (\underbrace{\rho \cdot A_1 \cdot ds_1}_{\text{mass}}) g y_1 + (\underbrace{\rho A_2 ds_2}_{\text{gain at the top}}) g y_2$$

loss from bottom

gain at the top

$$dK = \underbrace{-\frac{1}{2}(\rho \cdot A_1 ds_1) v_1^2}_{\text{loss at bottom}} + \underbrace{\frac{1}{2}(\rho A_2 ds_2) v_2^2}_{\text{gain at top}}$$

Incompressibility:

$$\rho A_1 ds_1 = \rho A_2 ds_2$$

$$A_1 ds_1 = A_2 ds_2$$

→ they all divide out!

$$P_1 - P_2 = \rho g (y_2 - y_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$$

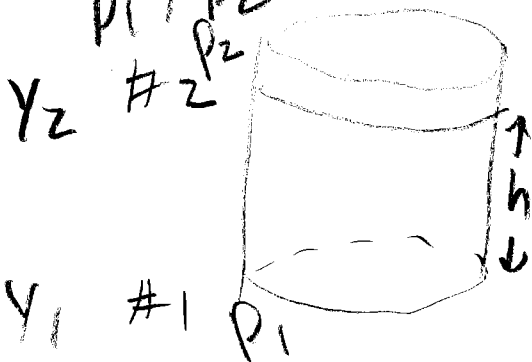
$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 = \text{constant (in flow tube)}$$

Examples:

$$v_1 = v_2 = 0$$

$$P_1 \neq P_2$$

$y_2 \neq y_1$



$$P_1 = P_2 + \rho g (y_2 - y_1)$$

$$h = y_2 - y_1$$

$$P_1 = P_2 + \rho g h$$