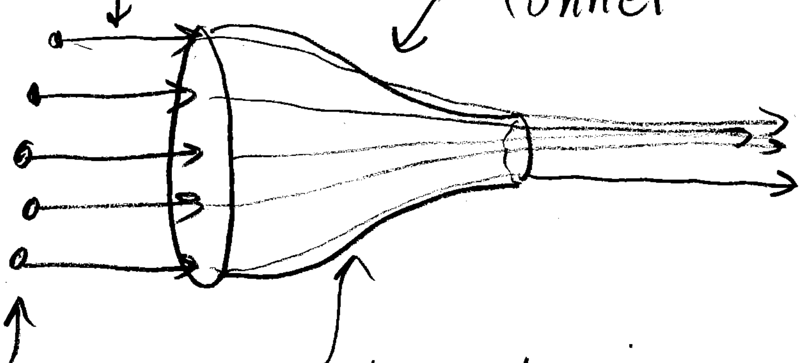
Fluid Flow

Steady: pattern does not change in time

velocities

"funnel"

higher velocities



specks of fluid

trajectories called "streamlines"
if not curly, "laminar flow"
"flow tubes" → follow a pre-defined set of specks

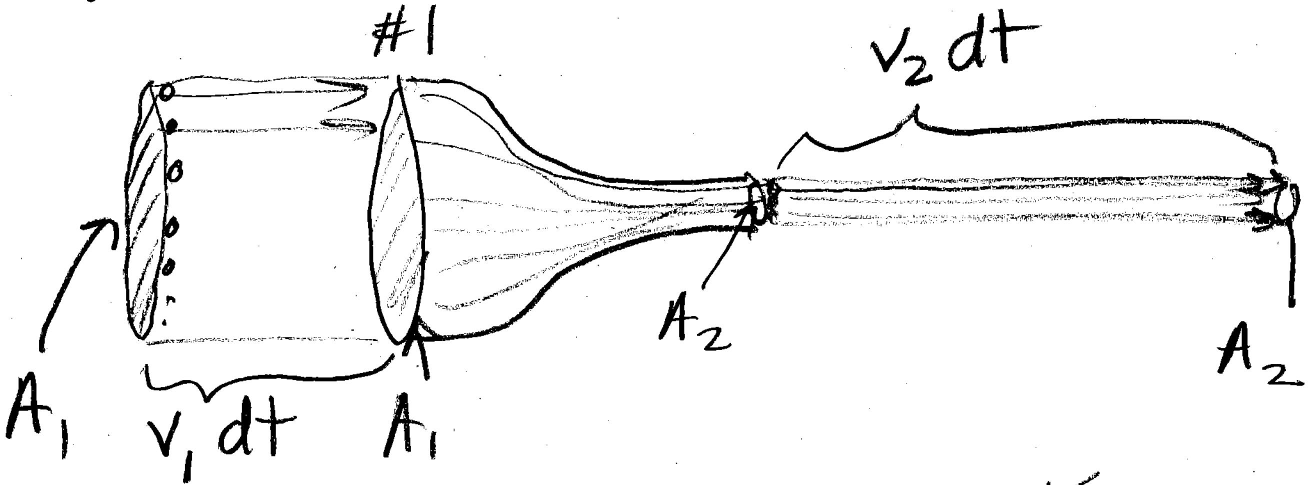
Whole thing above is a flow tube.

Continuity Equation

Like "Play-Do" what goes in must come out

really, # atoms is what $\propto$ mass

in time dt, compare volume in the flow tubes ...



<u>Mass</u>: $\rho_1 A_1 v_1 \cancel{dt} = \rho_2 A_2 v_2 \cancel{dt}$

$$(\rho_1 v_1) \times A_1 = (\rho_2 v_2) \times A_2$$

"continuity equation"

- true when $\vec{v} \perp$ to Area
- $\rho\vec{v} \equiv \vec{\phi}$ = "flux"
- $\frac{dV}{dt} = v \cdot A$ = volume change w/r to time.
- $\rho_1 = \rho_2$ for incompressible fluids

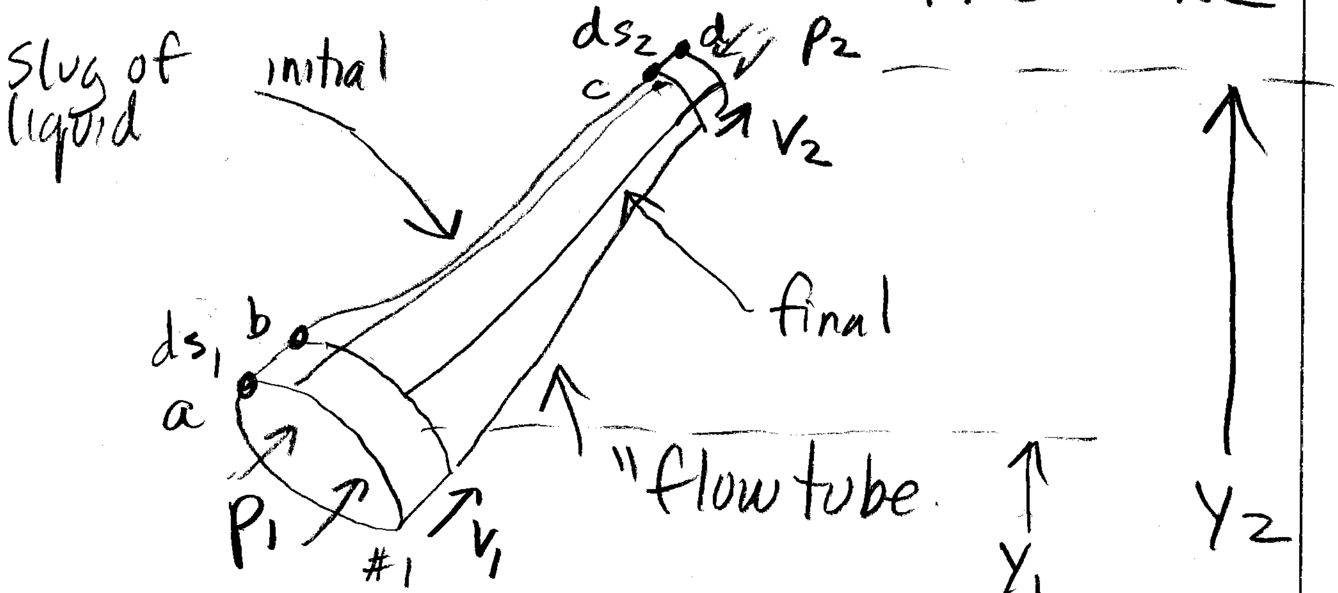
"Kinematics"

Bernoulli's Principal

→ apply energy considerations to some fluid.

→ surprise result:

FASTER = LESS PRESSURE



$$dW = P_1 A_1 ds_1 - P_2 A_2 ds_2$$

$$= dU + dK$$

↑ potential energy

↖ kinetic energy

assume incompressible

$$dU = \underbrace{-\underbrace{(\rho \cdot A_1 \cdot ds_1)}_{\text{mass}} g y_1}_{\text{loss from bottom}} + \underbrace{(\rho A_2 ds_2) g Y_2}_{\text{gain at the top}}$$

$$dK = \underbrace{-\frac{1}{2}(\rho \cdot A_1 ds_1) v_1^2}_{\text{loss at bottom}} + \underbrace{\frac{1}{2}(\rho A_2 ds_2) v_2^2}_{\text{gain at top}}$$

Incompressiblility:

$$\cancel{\rho} A_1 ds_1 = \cancel{\rho} A_2 ds_2$$

$$A_1 ds_1 = A_2 ds_2$$

→ they all divide out!

$$p_1 - p_2 = \rho g (y_2 - y_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

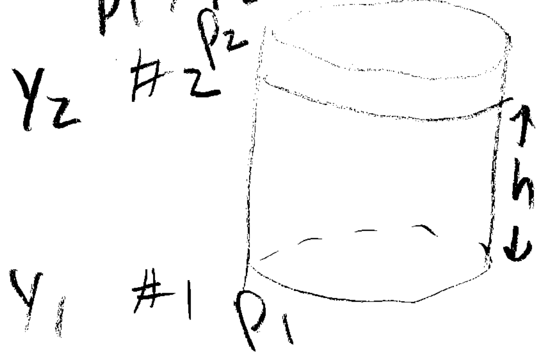
$$\boxed{p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = \text{constant (in flow tube)}}$$

Examples:

$$p_1 + \frac{1}{2}\cancel{\rho v_1^2} + \rho g y_1 = p_2 + \frac{1}{2}\underbrace{\cancel{\rho v_2^2}}_{0} + \rho g y_2$$

$v_1 = v_2 = 0$

$p_1 \neq p_2$



$$p_1 = p_2 + \rho g (y_2 - y_1)$$

$$h = y_2 - y_1$$

$$p_1 = p_2 + \rho g h$$