Physics 21 Problem Set 1

Harry Nelson

due Monday, January 15, In Class

Course Info: The instructor is Harry Nelson, the TA is Richard Eager. A web page for the course will be set up at http://hep.ucsb.edu/courses/ph21.

We meet MWF 10:00-10:50am in 1640 Broida. There are **two sections**, attendance at **both** is mandatory. Both sections are in 1508 Phelps, the first from 11:00 to 11:50am, the second from 3:00 to 3:50pm. In the sections, we will focus on problem solving.

The text for the course is 'An Introduction to Mechanics' by Kleppner and Kolenkow. (K&K). This is a hard textbook, but rewarding. Our plan is to cover much of the first six chapters of this text.

Working problems is crucial to the understanding of physics. Expect to spend at least 12 hours a week outside of class studying and working problems.

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

These problems pertain to the first three lectures, and the corresponding reading is pp. 1-13 of K&K.

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1. (a) Draw a 2-d coordinate system where y is North and x is East.
   (b) Vector $\mathbf{A}$ is a displacement from the origin in the Northeast direction by 3 meters. Draw this vector on your coordinate system.
   (c) Show the components A_x and A_y of $\mathbf{A}$ on your graph, and numerically evaluate these components.
   (d) Evaluate the components of $\hat{\mathbf{A}}$, a unit vector that is parallel to $\mathbf{A}$.
   (e) Repeat the last three steps for a second vector $\mathbf{B}$, which is a displacement from the origin in the South-Southwest direction by 2 meters. South-Southwest is halfway between the Southwest and South directions.
   (f) On your graph, show the vector $\mathbf{A}+\mathbf{B}$, using parallel translation and the tip-to-tail method.
   (g) Find and show $\mathbf{A}+\mathbf{B}$ using the components of the vectors.
   (h) Re-do the last two steps to find and show $\mathbf{A}-\mathbf{B}$.
   (i) Find the smaller angle θ between $\mathbf{A}$ and $\mathbf{B}$.
   (j) Evaluate $\mathbf{A}\cdot\mathbf{B}$, using the formula $AB\cos\theta$.
   (k) Do you get the same answer when you use the formula $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y$?
   (l) Evaluate the components of the vector that is the projection of $\mathbf{A}$ along the direction of $\mathbf{B}$.
   (m) Evaluate the components of the vector that is the projection of $\mathbf{B}$ along the direction of $\mathbf{A}$.
2. K&K, Problem 1.2
3. K&K, Problem 1.4

Some References to Calculus Texts

A very popular textbook is G. B. Thomas, Jr., "Calculus and Analytic Geometry," 4th ed., Addison-Wesley Publishing Company, Inc., Reading, Mass.

The following introductory texts in calculus are also widely used:
M. H. Protter and C. B. Morrey, "Calculus with Analytic Geometry," Addison-Wesley Publishing Company, Inc., Reading, Mass.
A. E. Taylor, "Calculus with Analytic Geometry," Prentice-Hall, Inc., Englewood Cliffs, N.J.
R. E. Johnson and E. L. Keokemeister, "Calculus With Analytic Geometry," Allyn and Bacon, Inc., Boston.

A highly regarded advanced calculus text is R. Courant, "Differential and Integral Calculus," Interscience Publishing, Inc., New York.

If you need to review calculus, you may find the following helpful: Daniel Kleppner and Norman Ramsey, "Quick Calculus," John Wiley & Sons, Inc., New York.

Problems

1.1 Given two vectors, $\mathbf{A} = (2\hat{\imath} - 3\hat{\jmath} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\imath} + \hat{\jmath} + 2\hat{\mathbf{k}})$, find: (*a*) $\mathbf{A} + \mathbf{B}$; (*b*) $\mathbf{A} - \mathbf{B}$; (*c*) $\mathbf{A} \cdot \mathbf{B}$; (*d*) $\mathbf{A} \times \mathbf{B}$.

Ans. (*a*) $7\hat{\imath} - 2\hat{\jmath} + 9\hat{\mathbf{k}}$; (c) 21

1.2 Find the cosine of the angle between

$$\mathbf{A} = (3\hat{\imath} + \hat{\jmath} + \hat{\mathbf{k}}) \qquad \text{and} \qquad \mathbf{B} = (-2\hat{\imath} - 3\hat{\jmath} - \hat{\mathbf{k}}).$$

Ans. -0.805

1.3 The direction cosines of a vector are the cosines of the angles it makes with the coordinate axes. The cosine of the angles between the vector and the x, y, and z axes are usually called, in turn α, β, and γ. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$, using either geometry or vector algebra.

1.4 Show that if $|\mathbf{A} - \mathbf{B}| = |\mathbf{A} + \mathbf{B}|$, then $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

1.5 Prove that the diagonals of an equilateral parallelogram are perpendicular.

1.6 Prove the law of sines using the cross product. It should only take a couple of lines. (*Hint*: Consider the area of a triangle formed by $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, where $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$.)