

(Trajectory)

(a)

$$\frac{1}{2}mv^2 = mg(z_0 - h)$$

$$v = \sqrt{2g(z_0 - h)}$$

vertical, horiz components:

$$v_{z\theta} = v \sin \theta \quad v_{x\theta} = v \cos \theta$$

vertical motion: top of trajectory is where z-component of velocity is zero happens at time t_1

$$0 = v_{z\theta} - gt_1 \quad t_1 = \frac{v_{z\theta}}{g}$$

$$\begin{aligned} \text{then } h' &= z' + v_{z\theta} t_1 - \frac{1}{2}gt_1^2 \\ &= h + \frac{v_{z\theta}^2}{g} - \frac{1}{2}g \frac{v_{z\theta}^2}{g^2} \end{aligned}$$

$$h' = h + \frac{v_{z\theta}^2}{2g} \quad \dots \text{ can also get from energy}$$

t_2 , time to drop to ground is then

$$\frac{1}{2}gt_2^2 = h' \Rightarrow t_2 = \sqrt{\frac{2h'}{g}}$$

and so, $d = v_{x\theta}(t_1 + t_2)$

$$d = v \cos \theta \left(\frac{v \sin \theta}{g} + \sqrt{\frac{2}{g} \left(h + \frac{v^2 \sin^2 \theta}{2g} \right)} \right)$$

$$d = \frac{v^2 \sin \theta \cos \theta}{g} + v \cos \theta \sqrt{\frac{2}{g} \left(h + \frac{v^2 \sin^2 \theta}{2g} \right)}$$

$$= \frac{2g(z_0 - h) \cdot \frac{1}{2}}{g} + \sqrt{2g(z_0 - h)} \cdot \frac{1}{\sqrt{2}} \sqrt{\frac{2}{g} \left(h + \frac{2g(z_0 - h) \cdot \frac{1}{2}}{2g} \right)}$$

$$= z_0 - h + \sqrt{2(z_0 - h) \left(h + \frac{1}{2}(z_0 - h) \right)}$$

$$= z_0 - h + \sqrt{2(z_0 - h) \cdot \frac{1}{2}(z_0 + h)}$$

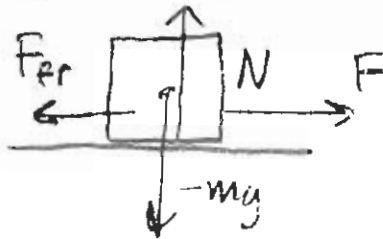
$$d = z_0 - h + \sqrt{z_0^2 - h^2}$$

(b) By inspection, d only grows smaller as h increases. You can reason this out by noting that the larger h gets, the smaller v gets, and so the smaller the horizontal component of v gets, and the shorter the time in the air becomes.

So, largest d occurs when $h=0$, and is $d=2z_0$

2.

(a)



$$N - mg = 0$$

$$N = mg = 1 \cdot 10 = 10 \text{ N}$$

$$ma_x = F - F_{fr}$$

$$4.8 \text{ N}$$

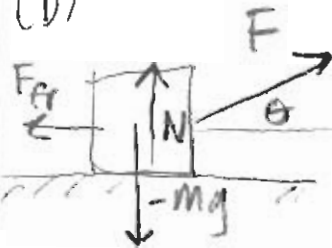
↑ maximum

$$\mu \cdot N = \frac{1}{2} \cdot 10 = 5 \text{ N}$$

Since $F < \mu N = 5 \text{ N}$,

you cannot move horizontally

(b)



$$N + F \sin \theta - mg = 0$$

$$0.34$$

$$N = mg - F \sin \theta = 1 \cdot 10 - 4.8 \cdot \sin(20^\circ)$$

$$N = 10 - 4.8 \cdot 0.34 = 10 - 1.64$$

$$N = 8.36 \text{ N}$$

$$ma_x = F \cos \theta - F_{fr}$$

maximum is

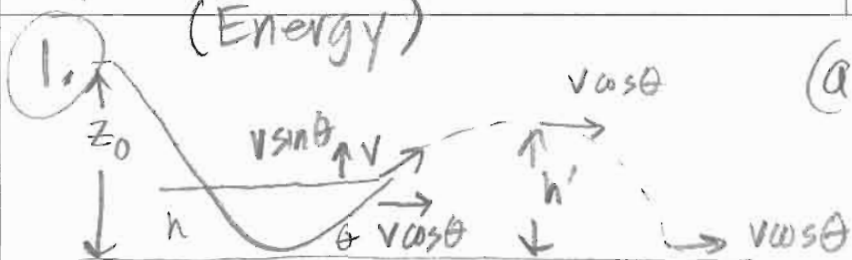
$$4.51 \text{ N}$$

$$4.8 \cdot \cos(20^\circ)$$

$$0.94$$

$$\mu \cdot N = \frac{1}{2} \cdot 8.36 = 4.18 \text{ N}$$

(Energy)



(a) at end of ramp

$$\frac{1}{2}mv^2 = mg(z_0 - h)$$

$$v = \sqrt{2g(z_0 - h)}$$

 $d \longrightarrow$

end of ramp to top of trajectory.

$$\frac{1}{2}mv^2 = mg(h' - h) + \frac{1}{2}mv^2 \cos^2 \theta$$

$$mg(z_0 - h) = mg(h' - h) + mg(z_0 - h) \cos^2 \theta$$

$$z_0 - (z_0 - h) \cos^2 \theta = h'$$

$$h' = z_0(1 - \cos^2 \theta) + h \cos^2 \theta$$

$$h' = z_0 \sin^2 \theta + h \cos^2 \theta$$

time to go from end of ramp to h' ... t_1
 time to go from h' to ground ... t_2

$$v \sin \theta - gt_1 = 0$$

$$t_1 = \frac{v}{g} \sin \theta$$

$$\frac{1}{2}gt_2^2 = h'$$

$$t_2 = \sqrt{\frac{2h'}{g}} = \sqrt{\frac{2}{g}(z_0 \sin^2 \theta + h \cos^2 \theta)}$$

$$d = v \cos \theta \cdot (t_1 + t_2) = \left(\frac{v^2}{g} \sin \theta \cos \theta + v \cos \theta \sqrt{\frac{2}{g}(z_0 \sin^2 \theta + h \cos^2 \theta)} \right)$$

$$d = \frac{2g(z_0 - h) \sin \theta \cos \theta}{g} + \sqrt{2g(z_0 - h)} \cos \theta \sqrt{\frac{2}{g}(z_0 \sin^2 \theta + h \cos^2 \theta)}$$

$$d = (z_0 - h) \cdot (2 \sin \theta \cos \theta) + \sqrt{2(z_0 - h)} \sqrt{2} \cos \theta \sqrt{2(z_0 \sin^2 \theta + h \cos^2 \theta)}$$

but $\sin \theta = \cos \theta = \frac{1}{\sqrt{2}}$

$$\text{so, } d = (z_0 - h) + \sqrt{(z_0 - h)(z_0 + h)} = z_0 - h + \sqrt{z_0^2 - h^2}$$

(b) Inspect... $h=0$ gives largest d , or

$$d = 2z_0$$

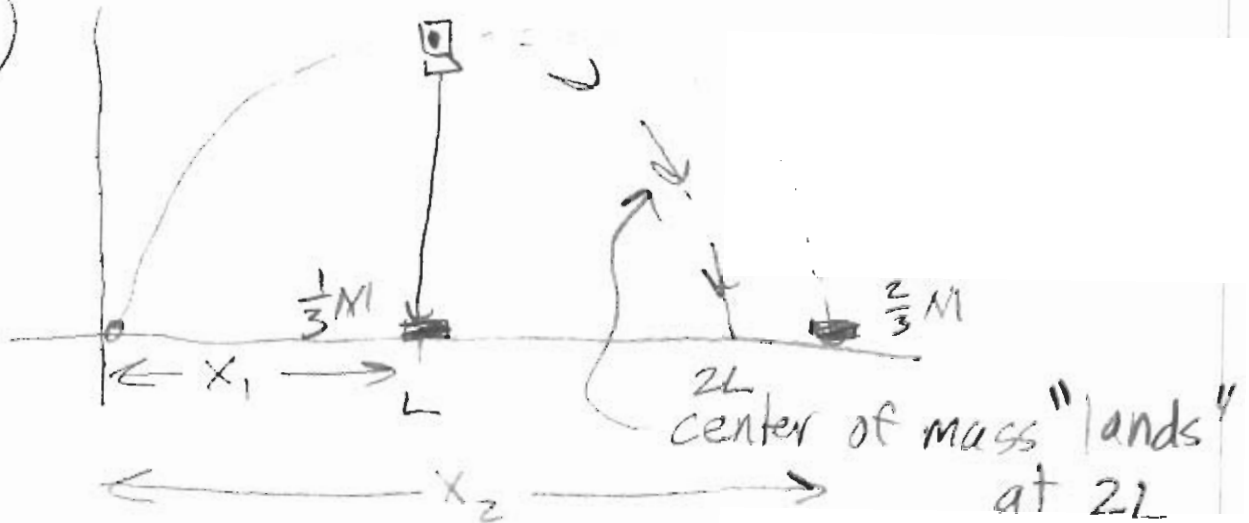
3.



$$-ma = -m \frac{v^2}{R} = -\mu mg$$

$$\mu = \frac{v^2}{gR} = \frac{20^2}{10 \cdot 20} = \frac{20}{10} = 2$$

4.



$$\frac{\frac{1}{3} M x_1 + \frac{2}{3} M x_2}{M} = 2L$$

$$x_1 = L$$

$$\frac{1}{3} L + \frac{2}{3} x_2 = 2L$$

$$\frac{2}{3} x_2 = \frac{5}{3} L$$

$$x_2 = \frac{5}{2} L$$

5.



(a) impulse =

$$mv_0 = F \Delta t \Rightarrow \Delta t = \frac{mv_0}{F} = \frac{4 \cdot 2}{10^3}$$

$$\Delta t = 8 \cdot 10^{-3} \text{ s}$$

$$(b) \frac{1}{2} k (\Delta x)^2 = \frac{1}{2} m v_0^2$$

$$\Rightarrow \Delta x = \sqrt{\frac{m}{k}} v_0 = \sqrt{\frac{4}{10^2}} \cdot 2$$

$$(c) T \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi \sqrt{\frac{m}{k}} = 2\pi \cdot \sqrt{\frac{4}{10^{-2}}}$$

$$T = 2\pi \cdot 2 \cdot 10^{-1} = 0.4\pi \text{ s}$$

It takes $\frac{1}{2}$ of a period to return.

$$t = T \cdot \frac{1}{2} = 0.2\pi \text{ s}$$

$$(6.) \frac{1}{2} m v^2 = -\frac{GMm}{R_e + h} - \left(-\frac{GMm}{R_e} \right)$$

note · $h = \frac{R_e}{2}$ $\frac{GM}{R_e^2} = g$ so

$$\frac{GM}{R_e} = g R_e$$

$$\frac{1}{2} v^2 = -\frac{GM}{\frac{3}{2} R_e} + \frac{GM}{R_e}$$

$$= \left(-\frac{2}{3} + 1 \right) g R_e$$

$$v^2 = \frac{2}{3} g R_e$$

$$v = \sqrt{\frac{2}{3} g R_e} = \sqrt{\frac{2}{3} \cdot 10 \cdot 64 \cdot 10^6}$$

$$v = 6530 \text{ m/s}$$



Intermediate: $V = \frac{m_1}{m_1 + m_2} u_1$

C.M.: $u_1 - V = u_1 - \frac{m_1}{m_1 + m_2} u_1 = \frac{m_2}{m_1 + m_2} u_1$

after scatter. m_1 has speed $-\frac{m_2}{m_1 + m_2} u_1$
in center of mass frame

in original frame: $v_1 = -\frac{m_2}{m_1 + m_2} u_1 + V$

$$= -\frac{m_2}{m_1 + m_2} u_1 + \frac{m_1}{m_1 + m_2} u_1$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 = f u_1$$

$$\frac{m_1 - m_2}{m_1 + m_2} = f$$

$$m_1 - m_2 = f m_1 + f m_2$$

$$(1 - f) m_1 = (1 + f) m_2$$

$$m_2 = \frac{1 - f}{1 + f} m_1$$

(b) $m_2 = \frac{1 - (-\frac{1}{2})}{1 - \frac{1}{2}} m_1 = \frac{3}{2} m_1 = 1.5 \text{ kg}$