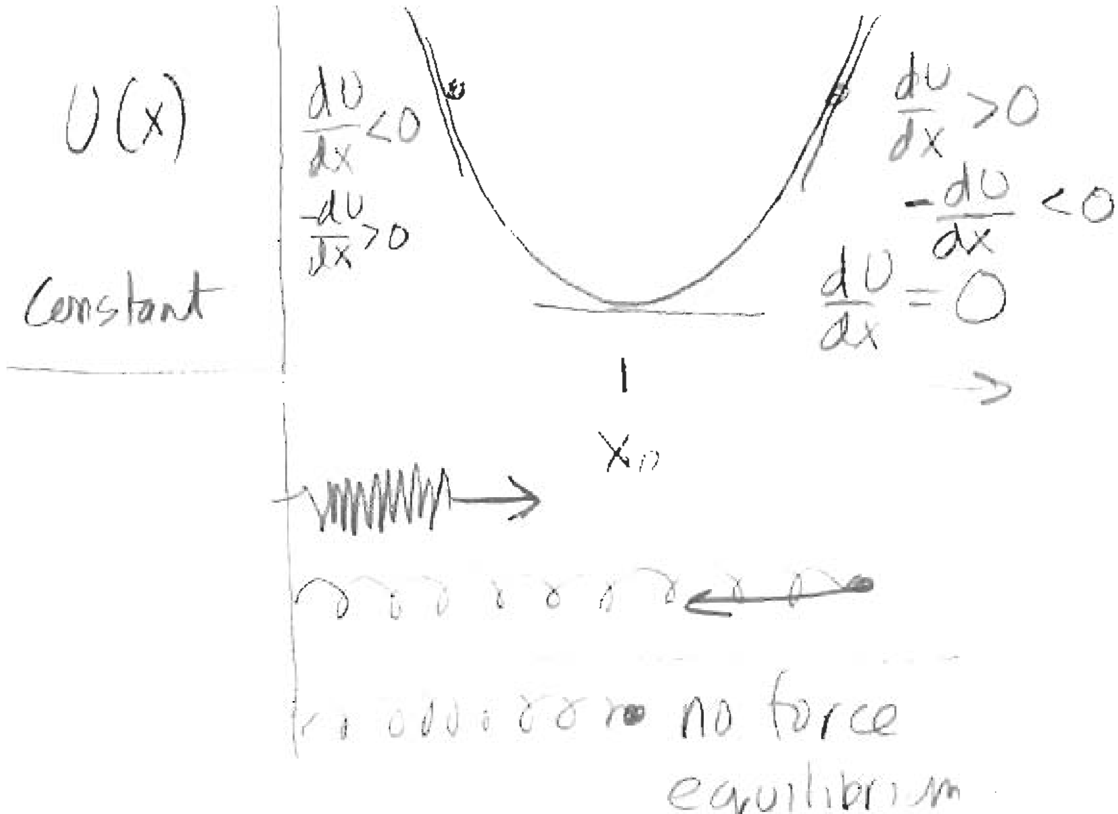
Spring $U(x) = \frac{1}{2}k(x-x_0)^2 + \text{constant}$

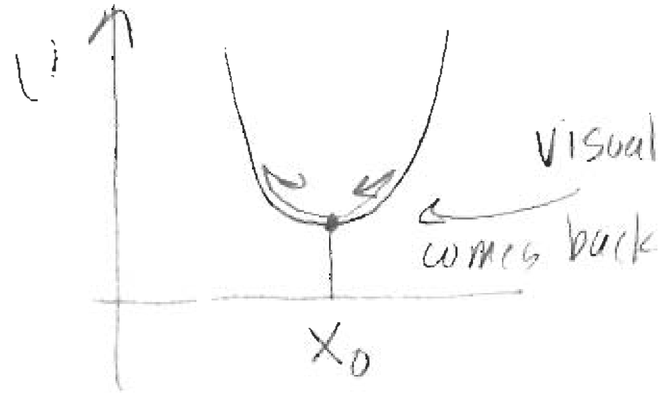


Points with $-\frac{dU}{dx} = 0$ are equilibrium points

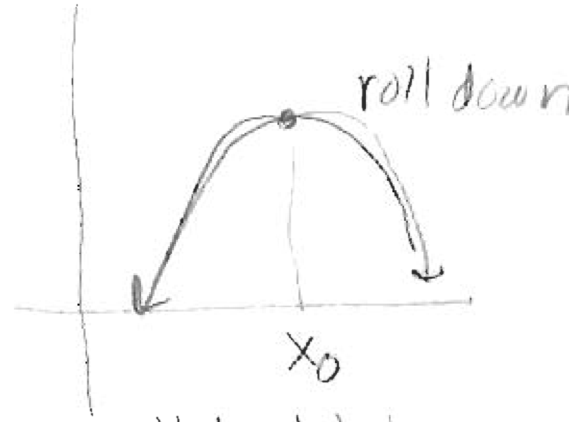
Why? $F = -\frac{dU}{dx} = 0$

Stable & Unstable

$\frac{d^2U}{dx^2} > 0$ $\quad$ $\frac{d^2U}{dx^2} < 0$



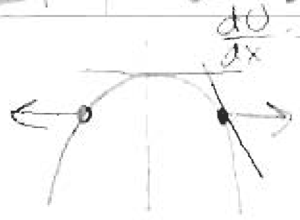
"restoring force"



"destabilizing force"

look at

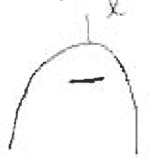
$\frac{dU}{dx} = 0$



$-\frac{dU}{dx}\Big|_a > 0$ ... positive force away from minimum

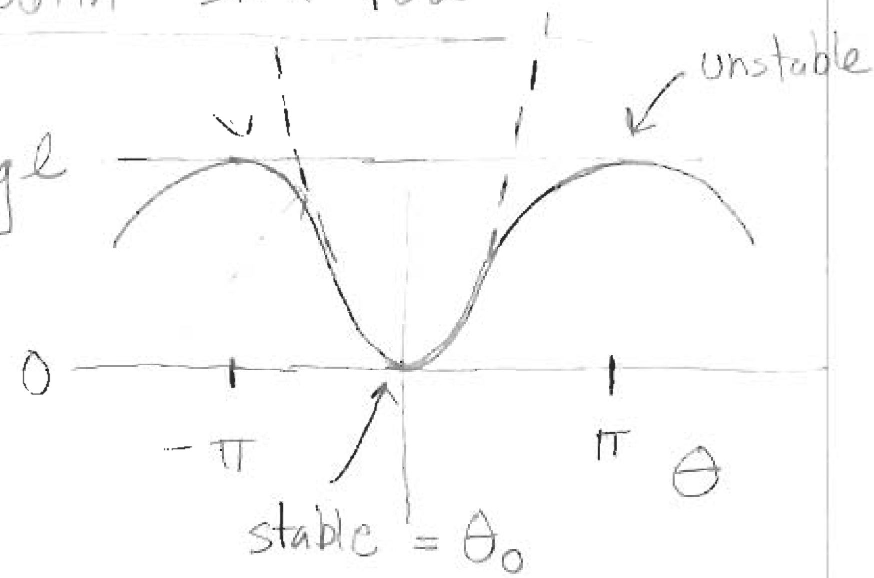
$-\frac{dU}{dx} < 0$ away from minimum
negative

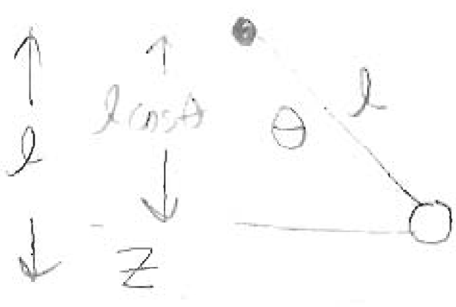
This time, moving away from point where $\frac{dU}{dx} = 0$ gives rise to force that pushes you away from $\frac{dU}{dx} = 0$ point



$\frac{d^2U}{dx^2}\Big|_\ell < 0$ ... local energy maximum
"unstable"

Pendulum with stiff rod





→ "parabolic approximation"

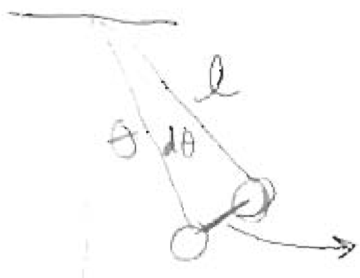
$z = \ell - \ell\cos\theta$

$= \ell(1-\cos\theta)$

$U(\theta) = mg\ell(1-\cos\theta)$

$U(z) = mgz$

$$U(\theta) \simeq U(\theta_0) + (\theta-\theta_0)\frac{dU}{d\theta}\Big|_{\theta=\theta_0} + \frac{1}{2}(\theta-\theta_0)^2\frac{d^2U}{d\theta^2}\Big| + (\text{ignore})$$



distance $= \ell \, d\theta$

speed $= \ell \dot{\theta}$

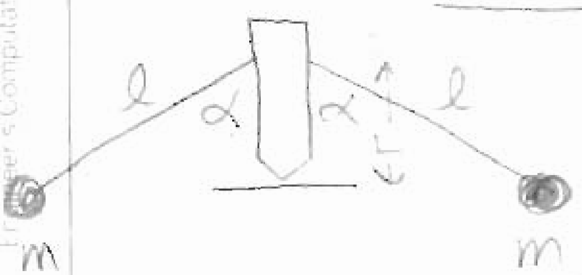
$$K = \frac{1}{2} m \cdot (\text{speed})^2 = \frac{1}{2} m \ell^2 \dot{\theta}^2, \quad B = m\ell^2$$

$$U = \frac{1}{2} m g \ell \theta^2, \quad A = mg\ell$$

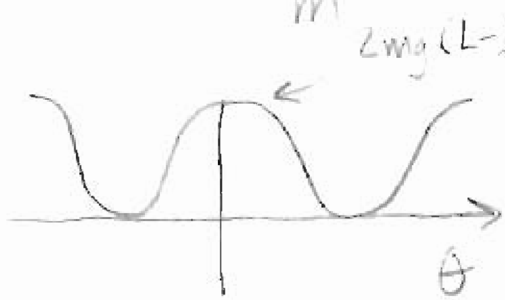
$$\boxed{\omega = \sqrt{\frac{A}{B}} = \sqrt{\frac{mg\ell}{m\ell^2}} = \sqrt{\frac{g}{\ell}}}$$

Can approximate, for small excursions about a stable equilibrium, any potential as a simple harmonic oscillator

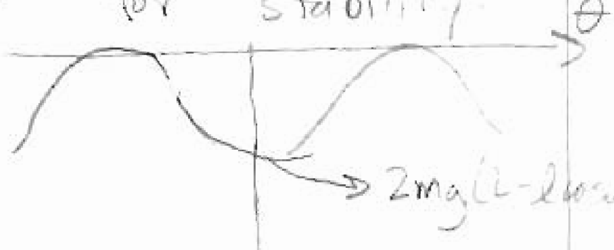
Teeter Toy (see p. 175)



$$U(\theta) = 2mg\cos\theta \underbrace{(L - l\cos\alpha)}_{\text{must be } - \text{ for stability}}$$



when $L - l\cos\alpha > 0$



when $L - l\cos\alpha < 0$

In either case.

$$\cos\theta \simeq \cos\theta_0 + (\theta-\theta_0)(-\sin\theta_0) + \frac{1}{2}(\theta-\theta_0)^2(-\cos\theta_0)$$

$$\theta_0 = 0 \quad \simeq 1 - \frac{1}{2}\theta^2$$

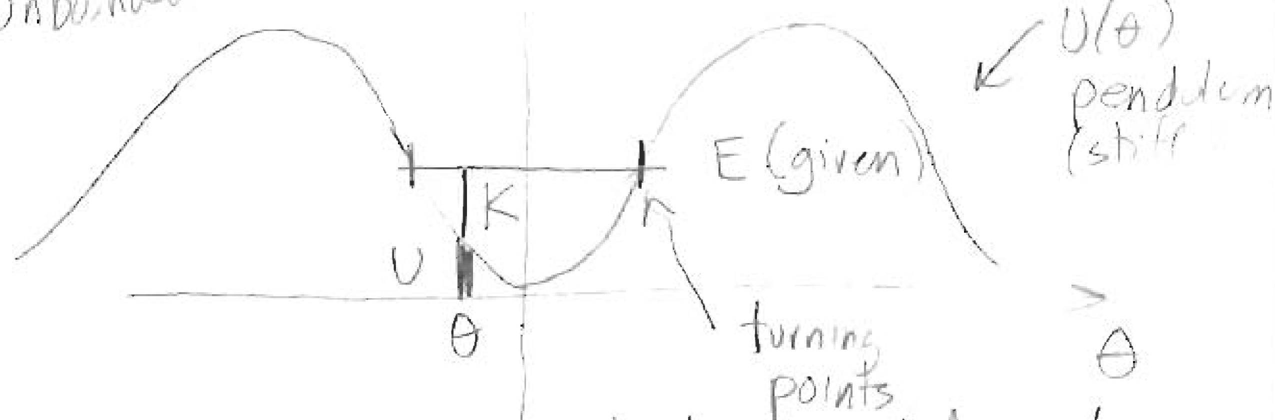
Pendulum: $U(\theta) \simeq mgl(1-\cos\theta) \simeq \frac{1}{2}mgl\,\theta^2$

Teeter Toy: $U(\theta) \simeq \underbrace{2mg(L-l\cos\alpha)}_{<0}\,(\underbrace{1}_{\text{constant}} - \frac{1}{2}\theta^2)$

Energy Diagram

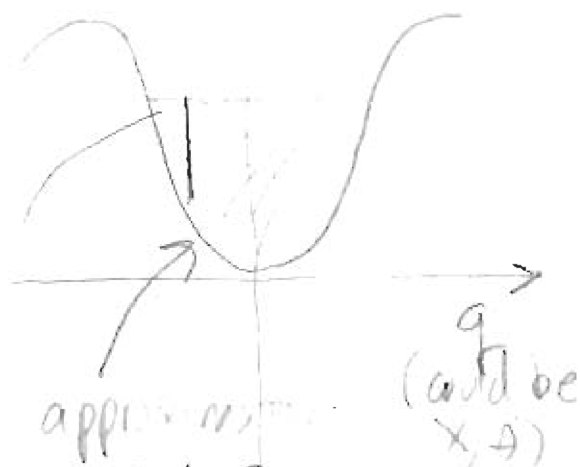
Since Total Energy E constant, following diagram graphically represents situation

Unbounded...



but $U+K=$ constant bounded motion

Small Oscillation



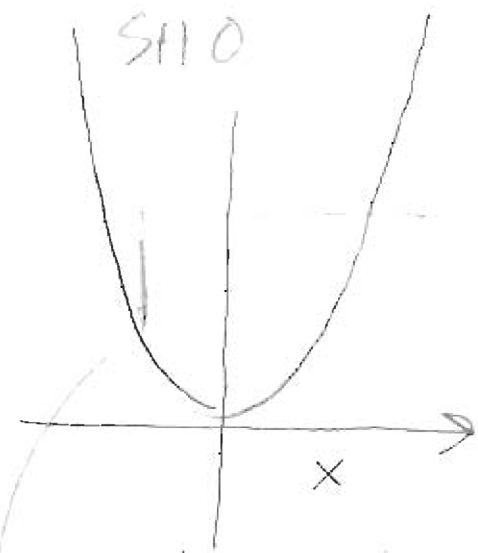
$$U=\frac{1}{2}Aq_r^2$$

$$K=\frac{1}{2}B\dot{q}_r^2$$

if you can do this, frequency of osc known!

$$\omega=\sqrt{\frac{B}{A}}$$

SHO



w/r to equilibrium

$$U=\frac{1}{2}kx^2$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$\omega=\sqrt{\frac{k}{m}}$$

STAEDTLER No. 937 811 Engineer's Computation Pad

4.11 non conservative forces - read

4.12 - actually non-conservative forces are situations where mechanical energy turns into heat

Heat ... energy put into random motion of particles (atoms)

M.E. → Heat happens

Heat → M.E. not really

Power

$$dW = \vec{F} \cdot d\vec{r}$$

$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$\frac{\text{Joules}}{\text{second}}$ = $\frac{\text{Newton-meter}}{\text{second}}$

Watts

746 Watts = 1 horse power

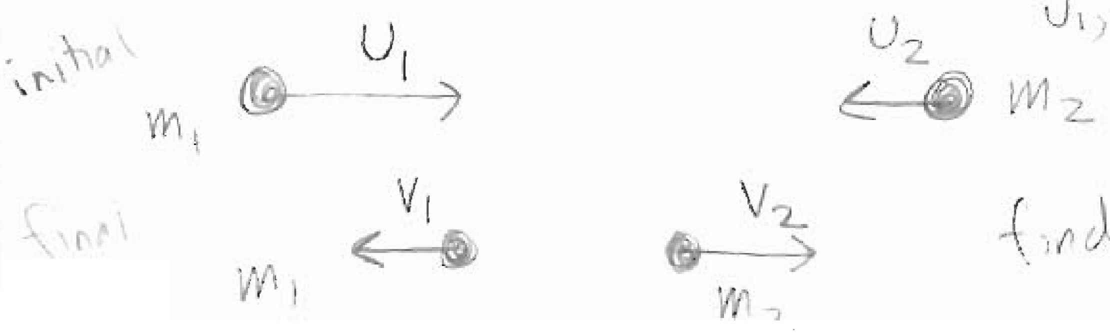
1 kilocalorie = 4200 Joules

diet = 2000-3000 kcal/day

1000 W/m² - sunlight on earth

Collisions (1-d, initially)

m_1, m_2
u_1, u_2 given



find v_1, v_2