Spring

$U(x) = \frac{1}{2}k(x - x_0)^2 + \text{constant}$

$\frac{dU}{dx} < 0$
$\frac{dU}{dx} > 0$
$
\frac{dU}{dx} = 0$

$x_0$

Points with $-\frac{dU}{dx} = 0$ are equilibrium points.

Why? $F = -\frac{dU}{dx} = 0$

Stable + Unstable

$\frac{d^2 U}{dx^2} > 0$
$\frac{d^2 U}{dx^2} < 0$
Visual coming back

"Restoring force"

Roll down

"Destabilizing force"
look at
\[ -\frac{du}{dx} < 0 \] negative
away from minimum
This time, moving away from point where \( \frac{du}{dx} = 0 \) gives rise to force that pushes you away from \( \frac{du}{dx} = 0 \) point
\[ \frac{du}{dx} \] local energy maximum "unstable"

Pendulum with stiff rod

\[ U(\theta) = U(\theta_0) + (\theta - \theta_0) \frac{du}{d\theta} \bigg|_{\theta = \theta_0} + \frac{1}{2} (\theta - \theta_0)^2 \frac{d^2u}{d\theta^2} \bigg|_{\theta = \theta_0} \]
distance = \ell d\theta 

speed = \ell \dot{\theta} 

K = \frac{1}{2} m \cdot speed^2 = \frac{1}{2} m \ell^2 \dot{\theta}^2, \quad B = m\ell^2 

U = \frac{1}{2} mg \ell \dot{\theta}^2, \quad A = mg\ell 

W = \sqrt{\frac{A}{B}} = \sqrt{\frac{mg\ell}{m\ell^2}} = \sqrt{\frac{g}{\ell}}
Can approximate for small excursions about a stable equilibrium any potential as a simple harmonic oscillator.

\[ U(\theta) = 2mgq \cos \theta (L - l \cos \theta) \]

Teeter Toy (see p. 175)

must be -

\[ 2mg(L - l \cos \theta) \]

for stability.

When \( L - l \cos \theta > 0 \)

In either case,

\[ \cos \theta = \cos \theta_0 + (\theta - \theta_0) (-\sin \theta_0) + \frac{1}{2} (\theta - \theta_0)^2 (-\cos \theta_0) \]

\[ \theta_0 = 0 \Rightarrow 1 - \frac{1}{2} \theta^2 \]

Pendulum: \[ U(\theta) = mgL(1 - \cos \theta) = \frac{1}{2} mgL \theta^2 \]

Teeter Toy: \[ U(\theta) = 2mg(L - l \cos \theta) (1 - \frac{1}{2} \theta^2) \]

\[ < 0 \]

Energy Diagram

Since Total Energy \( E \) constant, following diagram graphically represents situation.
Unbounded

but \( U + K = \text{constant} \)

Small Oscillation

approx. \( U = \frac{1}{2} A q^2 \)

\( K = \frac{1}{2} B q^2 \)

It you can do

This, frequency of osc known

\( w = \sqrt{ \frac{B}{A} } \)

w.r.to equilibrium

\( U = \frac{1}{2} k x^2 \)

\( K = \frac{1}{2} m x^2 \)

\( w = \sqrt{ \frac{k}{m} } \)
4.11 Non-conservative forces: Read

4.12 Actually, non-conservative forces are situations where mechanical energy turns into heat.

Heat... energy put into random motion of particles causes

\[ \text{M.E.} \rightarrow \text{heat happens} \]

\[ \text{heat} \rightarrow \text{M.E. not really} \]

**Power**

\[ dW = \vec{F} \cdot d\vec{r} \]

\[ \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v} \]

\[ \sqrt{\text{Joules per sec}} = \text{Newton-met} \]

\[ \sqrt{\text{Watts}} = \text{horses power} \]

746 W = 1 horsepower

1 kilowatt = 4200 Joules

diet = 2000-3000 kcal/day

1000 \( \text{W/m}^2 \) = sunlight on earth

**Collisions (1-d, initially)**

\[ m_1 \rightarrow U_1 \]

\[ m_1 \leftarrow V_1 \]

\[ U_2 \rightarrow m_2 \]

\[ m_2 \leftarrow V_2 \]

\[ \text{find } V_1, V_2 \]