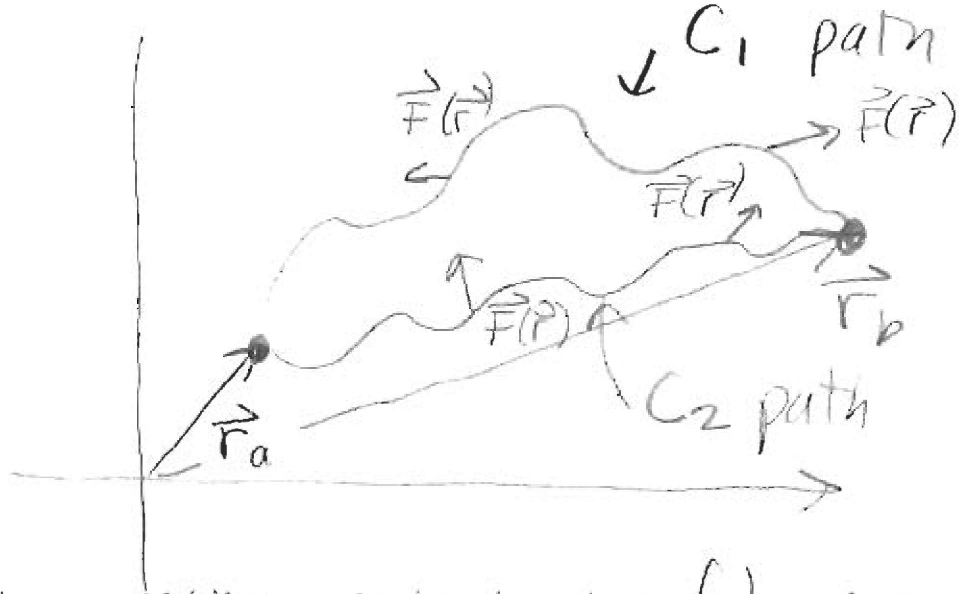
$$\frac{1}{2}m\left(v_x^2 + v_z^2\right) = \frac{1}{2}mv^2 \qquad v = |\vec{v}|$$

$$\vec{F} \cdot \underline{\underline{d\vec{r}}} = \vec{F} \cdot \underbrace{\left(d\underset{\sim}{x}\,\hat{\imath} + d\underset{\sim}{z}\,\hat{k}\right)}_{\text{called } d\vec{r}}$$

for a vecter $\vec{v}$ and $\vec{F}$

$$\frac{1}{2}m|\vec{v}_b|^2 - \frac{1}{2}m|\vec{v}_a|^2 = \oint_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) \cdot d\vec{r}$$

what is this C thing
It reminds that you need to specify how the mass goes from $\vec{r}_a$ to $\vec{r}_b$



But in many, many useful cases,

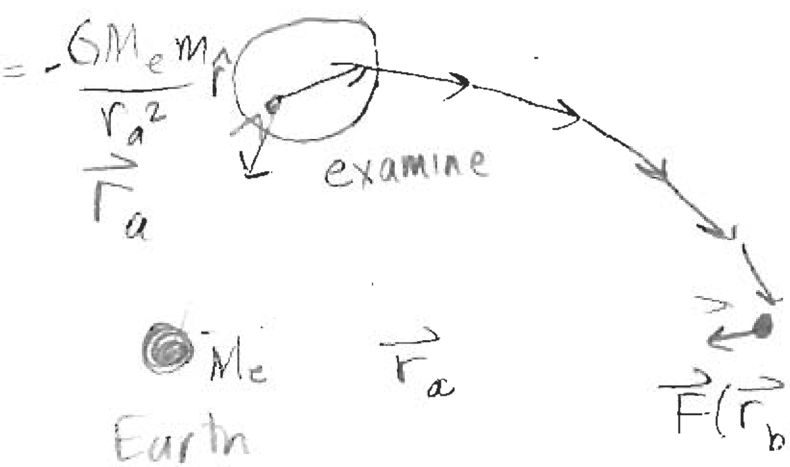
$$\int_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) \cdot d\vec{r} \text{ is independent of path!!}$$

Example #1 ... gravity near earth (look back)

Need concept of "Force Field"
Every point in space ... a force $\vec{F}$

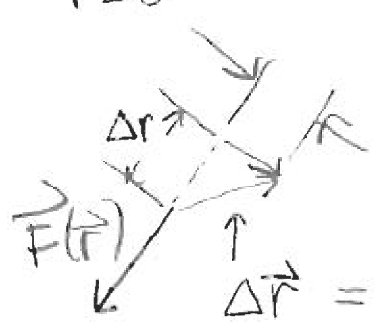
Example

$$\vec{F}(\vec{r}_a) = -\frac{GM_e m}{r_a^2}\hat{r}$$



$$\vec{F}(\vec{r}_b) = -\frac{GM_e m}{r_b^2}\hat{r}$$

$\Delta\theta$ & Δr describe the step $\vec{\Delta r}$



$\Delta\theta < 0$

$$\vec{\Delta r} = \Delta r\,\hat{r} + r\Delta\theta$$

$$\vec{F}(\vec{r}_a) \cdot \vec{\Delta r} \quad \left(-G\frac{M_e m}{r_a^2} \cdot \hat{r}\right)$$

$$\cdot\left(\Delta r\,\hat{r} + r\Delta\hat{\theta}\right)$$

$$dW = \vec{F}(\vec{r}_a) \cdot \Delta\vec{r} = -G\frac{M_e m}{r_a^2}\underbrace{\Delta r}_{\text{just radial step}}$$

$$\underset{\text{any } C!}{\int_{C,\,\vec{r}_a}^{\vec{r}_b}} \vec{F}(\vec{r})\,d\vec{r} = -GM_e m\int_{r_a}^{r_b}\frac{dr}{r^2}$$

$$= \underbrace{\frac{GM_e m}{r_a}}_{\text{function of } \vec{r}_a} - \underbrace{\frac{GM_e m}{r_b}}_{\text{function of } \vec{r}_b}$$

Doesn't happen for any arbitrary force ... friction is curve dependent.

When it does, note.

$$K_b - K_a = \underbrace{U(\vec{r}_a) - U(\vec{r}_b)}_{\text{potential energy}}$$