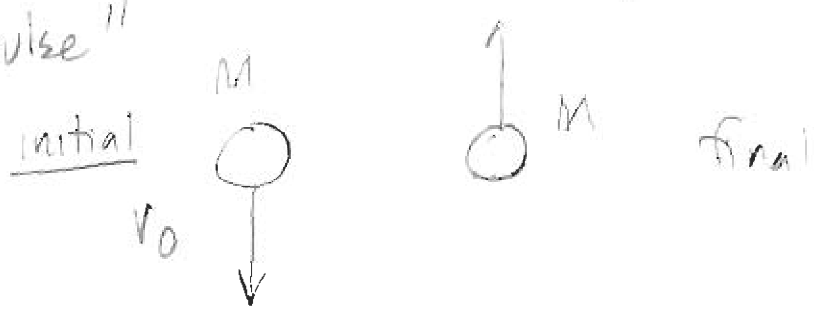
Impulse

$$\vec{F} = \frac{d\vec{P}}{dt}$$

point par

$$\underbrace{\int_0^t \vec{F}\,dt}_{\text{"Impulse"}} = \int_0^t \frac{d\vec{P}}{dt}\,dt = \underbrace{\vec{P}(t) - \vec{P}(0)}_{\text{change in momentum}}$$



$$\int_{t_0}^{t_0+\Delta t} \vec{F}\,dt = \vec{P}(\text{final}) - \vec{P}(\text{initial})$$

$$= +Mv_0\hat{k} - (-Mv_0\hat{k})$$

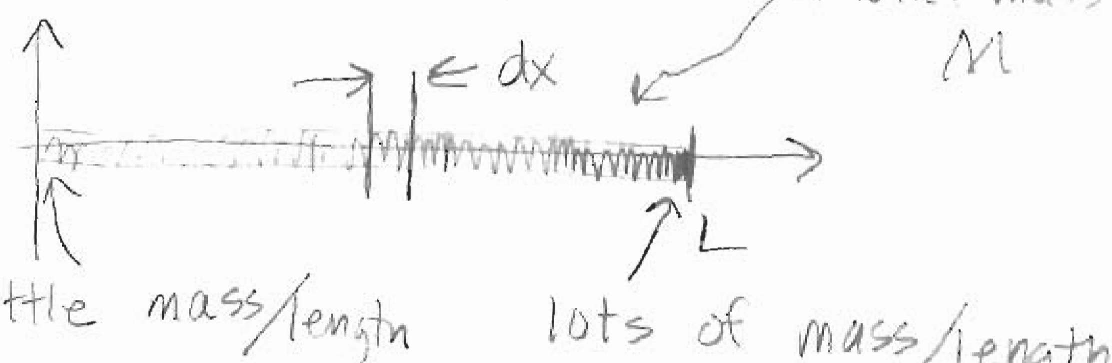
$$\therefore \vec{F}\cdot\Delta t = 2Mv_0\hat{k}$$

$$\vec{F} \simeq \frac{2Mv_0}{\Delta t}\hat{k}$$

Ball
$M = 0.2$ kg
$V_0 = 8$ m/s
$\Delta t \sim 10^{-3}$ s

$$|\vec{F}| \simeq \frac{2 \times 0.2 \times 8}{10^{-3}} \simeq 3.2 \cdot 10^{3}\ \text{N}$$

Non-uniform density



question: how much mass is in the slice dx?

$$dm = \lambda(x)\,dx$$

Two steps: "Normalization"

(1) $$\int dm = \int_0^L \lambda(x)\,dx = M$$

This step specifies constants.

(2) $$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\lambda(x)\,dx$$

example: $\lambda(x) \propto x$ (linear increase)

$$= \lambda_0 x$$

(1) Normalization

$$\int_0^L \lambda_0 x\,dx = \lambda_0 \frac{x^2}{2}\Big|_0^L = \frac{\lambda_0 L^2}{2}$$

$$\lambda_0 = \frac{2M}{L^2}$$

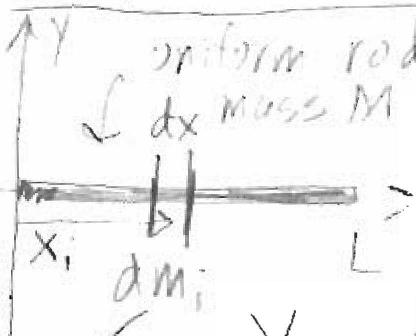
$$|\vec{r}_1{}'| = \frac{m_2}{m_1+m_2}|\vec{r}_1-\vec{r}_2| = \frac{m_2}{m_1+m_2}\ell, \propto m_2$$

$$|\vec{r}_2{}'| = \frac{m_1}{m_1+m_2}|\vec{r}_1-\vec{r}_2| = \frac{m_1}{m_1+m_2}\ell, \propto m$$

these are proportional to the other, so the $|\vec{r}_i|$ to the big mass will be small, and vice versa.

When thrown, the center of mass follows the parabolic trajectory.

Continuous mass distributions



← "obviously"

$$X = \frac{L}{2} \quad Y = 0 \quad Z = 0$$

$$X = \frac{1}{M}\cdot\sum x_i\, dm_i \Rightarrow \frac{1}{M}\int_0^L x\frac{M}{L}\cdot dx$$

$$dm_i = \frac{M}{L}\cdot dx$$

$$= \frac{1}{L}\int_0^L x\,dx = \frac{1}{L}\frac{1}{2}x^2\Big|_0^L = \frac{1}{2}L$$