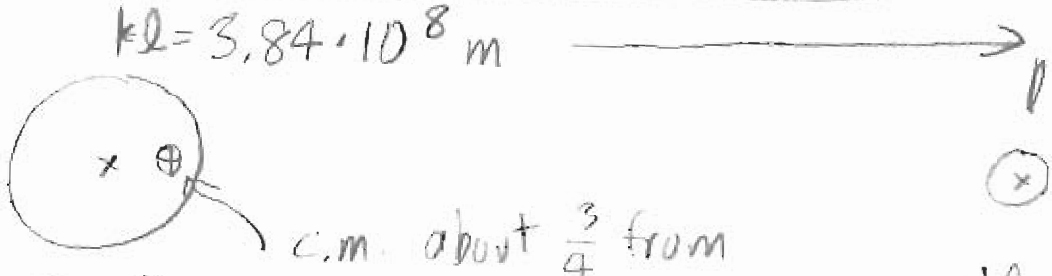
The Earth and the Moon



c.m. about $\frac{3}{4}$ from earth center to surface

Earth
$M_e = 5.98 \cdot 10^{24}$ kg
$R_e = 6.37 \cdot 10^6$ m

Moon
$M_m = 7.34 \cdot 10^{22}$ kg
$R_m = 1.74 \cdot 10^6$ m

Center of mass

$$\vec{R}_{cm} = \frac{M_e \cdot \vec{r}_e + M_m \vec{r}_m}{M_e + M_m}$$

(as if all mass concentrated at center when symmetric about center)

Choose smart origin: on center of earth!

$$\vec{R}_{cm} = \frac{M_m}{M_e + M_m} \vec{r}_m$$ → points from earth to moon

$$= \frac{M_m / M_e}{1 + M_m / M_e} \cdot \left(\vec{r}_m = \underline{l} \, \hat{x} \right)$$

↳ $3.84 \cdot 10^8$ m

$$\frac{M_m}{M_e} = \frac{7.34 \cdot 10^{22}}{5.98 \cdot 10^{24}} \frac{\text{kg}}{\text{kg}} = \frac{1}{81.5}$$

$$\vec{R}_{cm} = \frac{1/81.5}{1 + 1/81.5} \times 3.84 \cdot 10^8 \text{ m} \cdot \hat{x} = \frac{1}{82.5} \cdot 3.84 \cdot 10^8 \text{ m } \hat{x}$$

$$= 4.65 \cdot 10^6 \text{ m } \hat{x}$$

$$\frac{|\vec{R}_{cm}|}{R_e} = \frac{4.65 \cdot 10^6 \text{ m}}{6.37 \cdot 10^6 \text{ m}} = 0.731$$

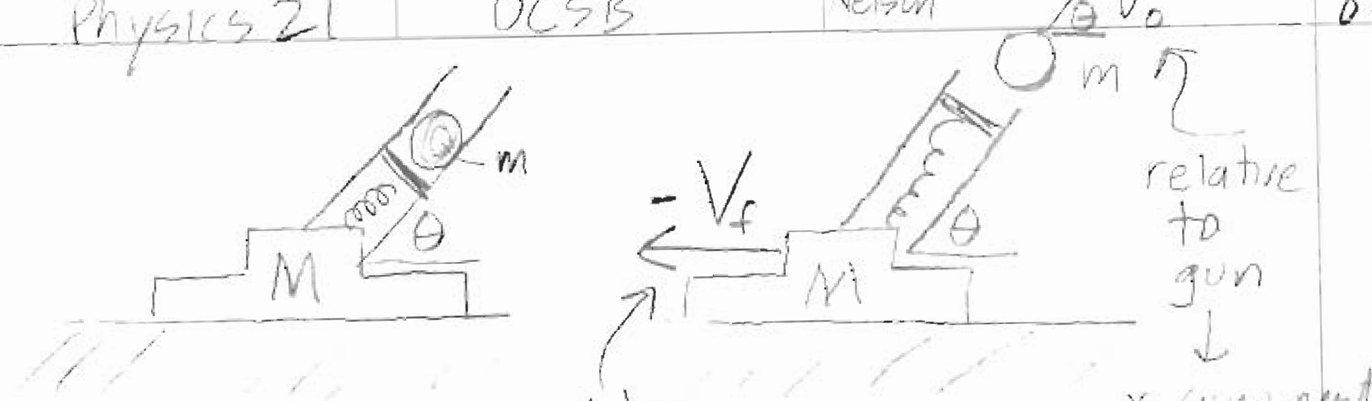


Table
Initial

relative to table

Final

x component relative to table is $V_0\cos\theta - V_f$ (computed earlier)

Vertical: $\sum F_{ext,y} \neq 0$, unless you include the earth in the system

$\sum F_{ext,x} = 0$ however (spring is an internal force)

$\frac{dP_x}{dt} = 0 \Rightarrow P_x = \text{constant}$

Initial: $P_x = M\cdot 0 + m\,0 = 0$

Final: $-MV_f + m(v_0\cos\theta - V_f) = 0$

$$mv_0\cos\theta = (M+m)V_f$$

$$V_f = \frac{m}{M+m}v_0\cos\theta \quad \to 0 \text{ as } M\to\infty$$

$$v_0\cos\theta - V_f = \frac{(M+m)v_0\cos\theta}{(M+m)} - \frac{mv_0\cos\theta}{M+m}$$

$$= \frac{M}{\ }\, V_0\cos\theta \quad \to V_0\cos\theta$$

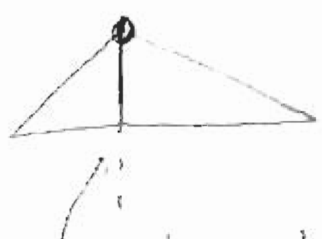
$$\lambda(x) = 2\,\frac{M}{L}\cdot\frac{x}{L}$$

(2) $$x_{cm} = \frac{1}{M}\int_0^L x\,\lambda(x)\,dx = \frac{1}{M}\int_0^L x\cdot 2\,\frac{M}{L}\,\frac{x}{L}\,dx$$

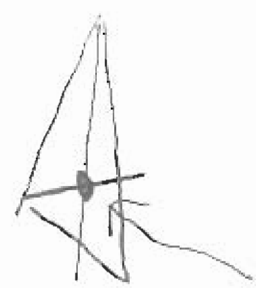
$$= \frac{2}{L^2}\int_0^L x^2\,dx = \frac{2}{3L^2}\,x^3\Big|_0^L$$

$$\boxed{x_{cm} = \frac{2}{3}L}$$

<u>2-d</u>: best way is to hang + intersect



c.m. along line.



(3-d

C.M.

<u>Conservation of Momentum</u>

<u>System</u>: $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$

then $$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext,i}$$

what happens when $\sum \vec{F}_{ext,i} = 0$

$$\frac{d\vec{P}}{dt} = 0$$ (may happen for only 1 component