Concept of Momentum

\[ \vec{F}_{\text{Net}} = m \vec{a} = m \frac{d^2 \vec{r}}{dt^2} = m \vec{\omega} \]

when \( m \) is constant as a function of time, then

\[ m \frac{d^2 \vec{r}}{dt^2} = \frac{d}{dt} (m \frac{d \vec{r}}{dt}) = \frac{d}{dt} (m \vec{v}) \]

Momentum: \( \vec{p} = m \vec{v} \) (definition)

Why? Some situations, \( \vec{p} \) easier to work with... "deeper"

then, \( \vec{F}_{\text{Net}} = \frac{d \vec{p}}{dt} \leq \text{true even in relativity} \)

Particularly useful for extended bodies... not point particles.

Consider an extended body as a big throng of point particles, key concept: net internal force is zero... Newton's Third Law.
Categorize forces: internal vs. external

When adding up forces on #1
\[ \frac{\Delta \vec{p}_{i}}{\Delta t} = \sum_{j} \vec{F}_{ij} = \sum_{j} \vec{F}_{\text{internal},j} + \vec{F}_{\text{ext},j} \]
include all forces, internal + external

\[ \frac{\Delta \vec{p}_{2}}{\Delta t} = \sum_{i} \vec{F}_{i2} = \sum_{i} \vec{F}_{\text{internal},i} + \vec{F}_{\text{ext},i} \]

Add up all
\[ \sum \frac{\Delta \vec{p}_{ij}}{\Delta t} = \sum_{i,j} \vec{F}_{ij} = 0 + \sum_{i,j} \vec{F}_{\text{ext},ij} \]

Internal forces cancel in pairs
by Newton's 3
\[
\sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_{ext}, \quad \text{can neglect internal forces}
\]

**Flying "bola"**

\[
\frac{d}{dt} (\vec{p}_1 + \vec{p}_2 + \vec{p}_3) = m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + m_3 \frac{d\vec{v}_3}{dt}
\]

Neglect string!

\[
\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \quad \frac{d\vec{p}}{dt} = (m_1 + m_2 + m_3) \vec{a} = M \vec{a}
\]

Motion (\(\vec{p}_i(t)\))

motion of \(\sum \vec{p}_i\) is that of a single particle, mass \(M\)

Center of mass

Trajectory of precisely what follows that of something with mass \(M\)?

Find an \(\vec{R}\) such that

\[
\vec{F} = M \vec{R} = \frac{d}{dt} \sum m_i \vec{v}_i = \frac{d}{dt} \sum m_i \vec{v}_i = \sum m_i \frac{d\vec{v}_i}{dt}
\]

assume \(\frac{dm_i}{dt} = 0\)
\[ \mathbf{R} = \sum m_i \mathbf{r}_i \]
\[ \mathbf{R} = \frac{1}{M} \sum m_i \mathbf{r}_i = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} \]

"center of mass"

"m-weighted mean position"

Example: baton: two masses joined by thin rod, infinitesimal mass center of mass is: 1) on a line between
2) closer to the heavier mass

Formula: \[ \mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} \]

\( \mathbf{P} \) to bigger one, has bigger coefficient \( \lim_{m_1 \to \infty} \mathbf{R} = \mathbf{R}_1, \lim_{m_2 \to \infty} \mathbf{R} = \mathbf{R}_2 \)

\[ \mathbf{r}_1' = \mathbf{r}_1 - \mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_1 - m_1 \mathbf{r}_1 - m_2 \mathbf{r}_2}{m_1 + m_2} = \frac{m_2}{m_1 + m_2} (\mathbf{r}_1' - \mathbf{r}_2) \]

\[ \mathbf{r}_2' = \mathbf{r}_2 - \mathbf{R} = \frac{m_1 \mathbf{r}_2 + m_2 \mathbf{r}_2 - m_1 \mathbf{r}_1 - m_2 \mathbf{r}_2}{m_1 + m_2} = - \frac{m_1}{m_1 + m_2} (\mathbf{r}_1' - \mathbf{r}_2) \]

both \( \mathbf{r}_1' \neq \mathbf{r}_2' \)