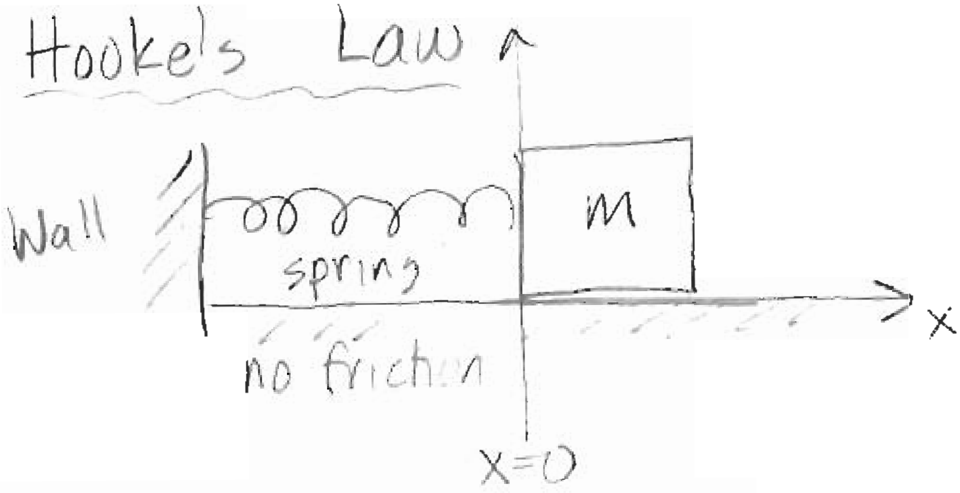
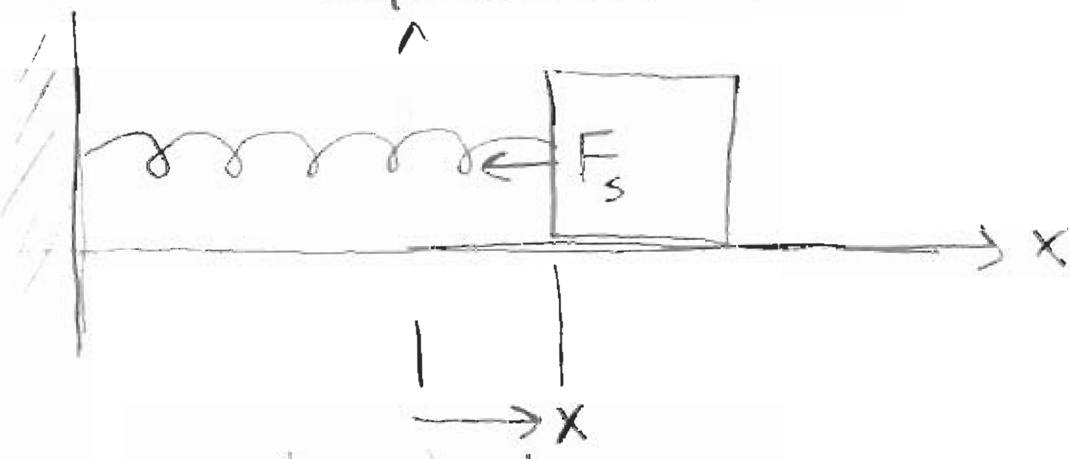
First Look at Simple Harmonic Oscillator

Hooke's Law



Say, when m is at x=0, it feels no net horizontal force. this is called the equilibrium point



F_s proportional to $-x$

"restoring" force

linear

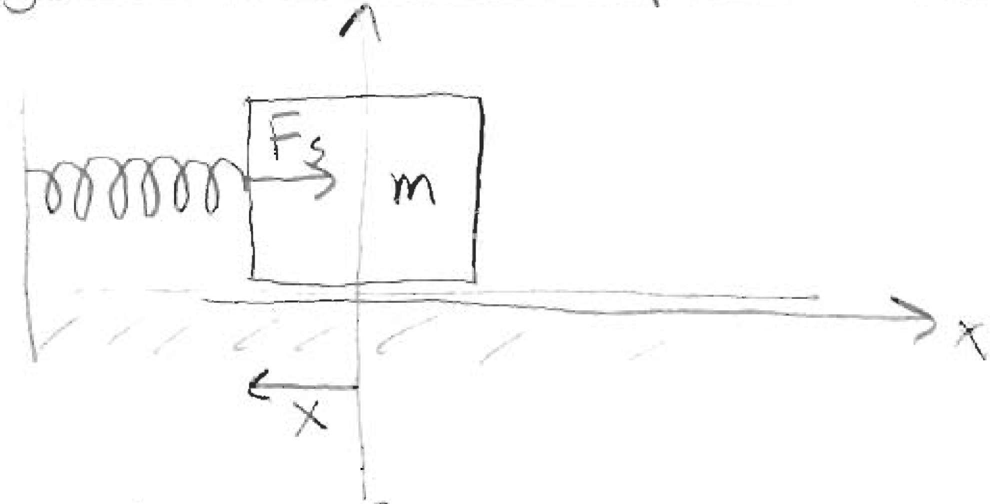
$$F_s = -kx$$

Newtons

meters

$\frac{\text{Newtons}}{\text{meter}}$

note: when x (displacement) is negative, F_s is positive



when no other forces...

$$m\ddot{x} = -kx$$

$$\boxed{m\frac{d^2x}{dt^2} = -kx}$$ differential equation

① Second order $\underbrace{\text{two}}$ arbitrary constants in solution!

recall: $m\frac{d^2x}{dt^2} = 0$, $x = x_0 + v_0 t$

second order (two derivatives)

two constants!

② Just remember

$x(t) = \cos t$ or $\sin t$

$\dot{x}(t) = -\sin t$ or $\cos t$

$\ddot{x}(t) = -\cos t$ or $-\sin t$

hmm... $\ddot{x}(t) = -x(t)$ either way

CLOSE

want... $\ddot{x} = -\frac{k}{m} x$

think chain rule!

try: $x(t) = A \sin\left(\sqrt{\frac{k}{m}}\, t\right) + B \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

$$\dot{x}(t) = A \cos\left(\sqrt{\frac{k}{m}}\, t\right) \cdot \sqrt{\frac{k}{m}} - B \sin\left(\sqrt{\frac{k}{m}}\, t\right) \cdot \sqrt{\frac{k}{m}}$$

$$\ddot{x}(t) = -A \sin\left(\sqrt{\frac{k}{m}}\, t\right) \cdot \frac{k}{m} - B \cos\left(\sqrt{\frac{k}{m}}\, t\right) \cdot \frac{k}{m}$$

$$= -\frac{k}{m}\left(A \sin\left(\sqrt{\frac{k}{m}}\, t\right) + B \cos\left(\sqrt{\frac{k}{m}}\, t\right)\right)$$

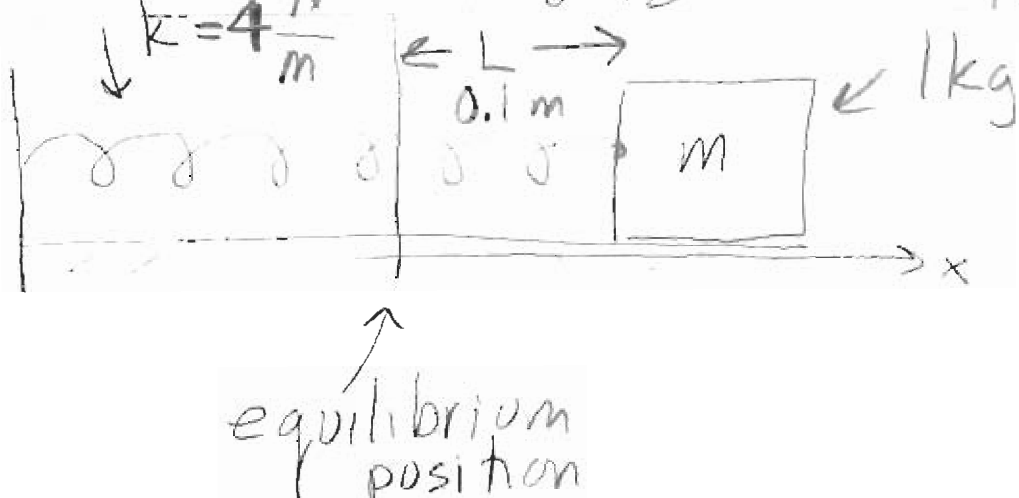
$$\ddot{x}(t) = -\frac{k}{m}\, x(t)$$

tired of writing, call $\boxed{\omega = \sqrt{\frac{k}{m}}}$

so, $x(t) = A \sin(\omega t) + B \cos(\omega t)$

constants determined by $x(0)$, $\dot{x}(0)$, usually

$\omega = \sqrt{\frac{k}{m}}$ ← spring ← mass



pull to L, release, from rest

→ get $x(t)$, find maximum velocity

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{1}} = 2\ \frac{1}{s}$$

$$\sqrt{\frac{N/m}{kg}} = \sqrt{\frac{kg\cdot\frac{m}{m}\frac{1}{s^2}}{kg}} \quad \text{check} \quad = \sqrt{\frac{1}{s^2}} = \frac{1}{s}$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

$$x(0) = L = A\sin(\omega\cdot 0) + B\cos(\omega\cdot 0)$$

$$\boxed{L = B}$$

$$\dot{x}(t) = \omega A\cos(\omega t) - \omega B\sin(\omega t)$$

$$\dot{x}(0) = \omega A\cos(\omega\cdot 0) - \omega B\sin(\omega\cdot 0)$$

$$0 = \omega A \Rightarrow \boxed{A = 0}$$

from rest

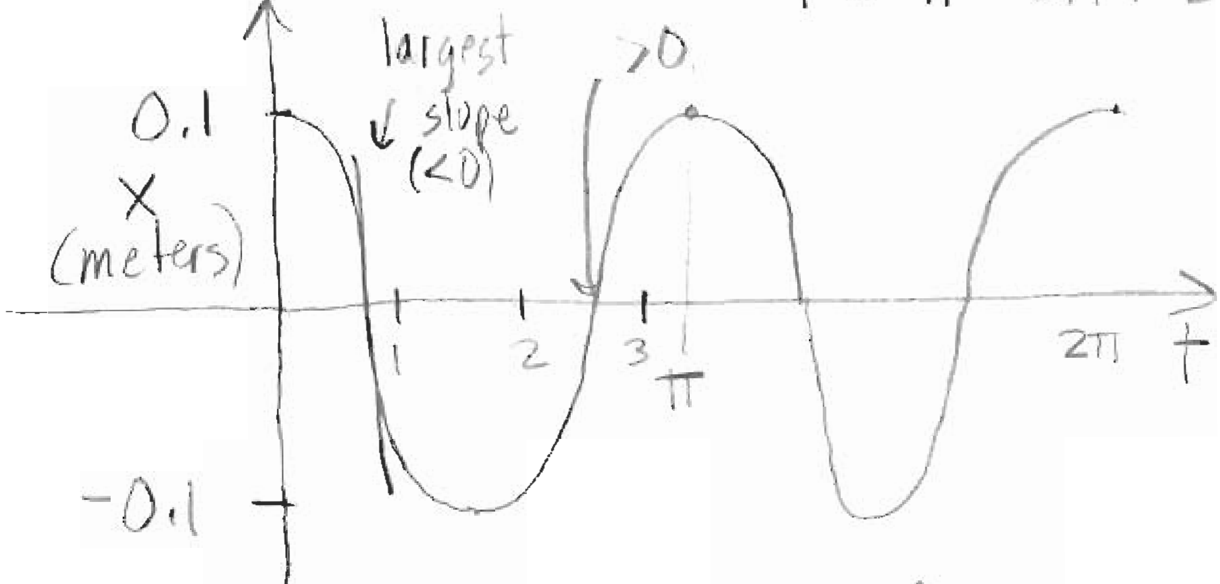
$$x(t) = 0 \cdot \sin(2 \cdot t) + L\cos(2 \cdot t)$$

$$\underset{0.1}{\uparrow}$$

$$\boxed{x(t) = 0.1\cos(2t)}$$

period, $2 \cdot T = 2\pi$

$T = \pi = 3.14$ s



$$v(t) = \dot{x}(t) = 0.1\,(-\sin(2t)) \times 2$$

$$= -0.2\,\sin(2t)$$