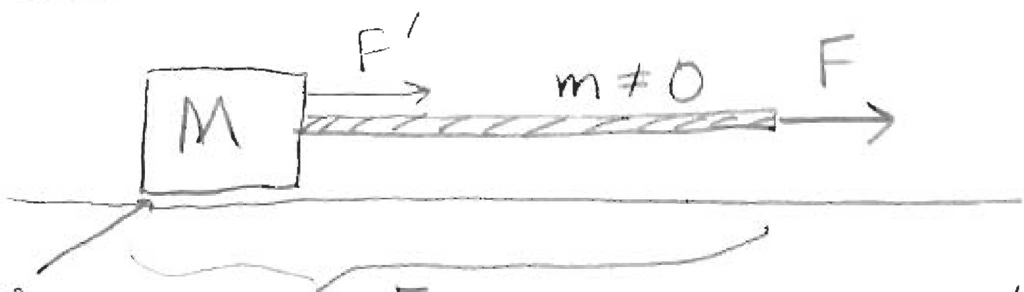
$$\mu \cdot m\omega^2 R - mg > 0$$

$$\omega^2 > \frac{g}{\mu R}$$

$$\boxed{\omega > \sqrt{\frac{g}{\mu R}}}$$

Real Ropes



no friction

$$a = \frac{F}{m+M} \qquad \therefore\ Ma = F'$$

$$\frac{M}{m+M} F = F'$$

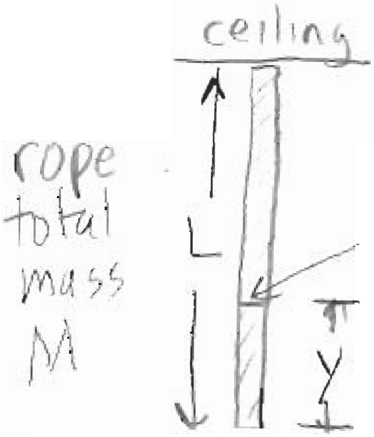
now look at rope alone . .

$$ma = F - F' = F - \frac{M}{m+M} F$$

$$ma = \left(\frac{m+M}{m+M} - \frac{M}{m+M}\right) F = \frac{m}{m+M} F$$

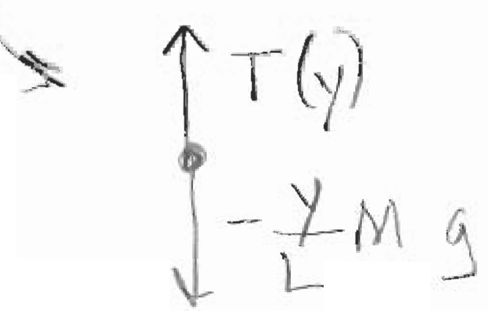
$$a = \frac{F}{m+M} \checkmark$$

Introducing Integral Calculus



← how much mass does rope in length x have?

$$= \frac{x}{L} \cdot M$$



$$T(y) - \frac{y}{L} Mg = 0$$

$$\boxed{T(y) = \frac{y}{L} Mg}$$

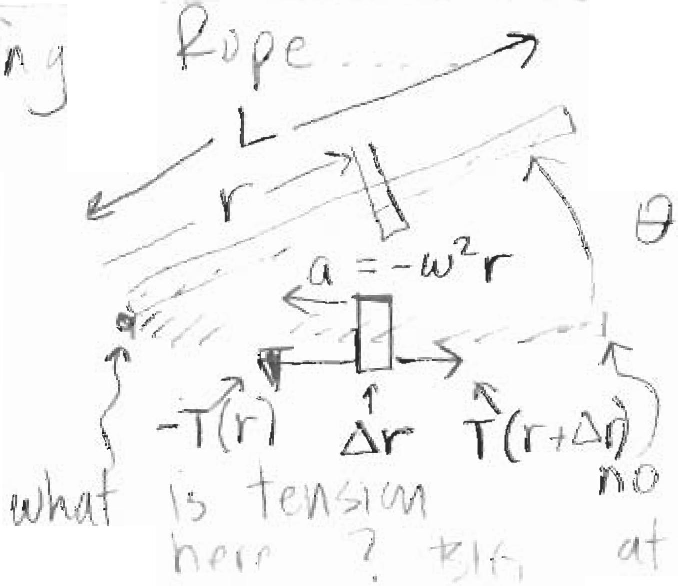
$y \to 0$ (rope end), $T = 0$

$y \to L$ (rope at ceiling)

Whirling Rope...

mass M

$\theta = \omega t$



$$T(r+\Delta r) - T(r) = -\left(\text{mass in } \Delta r\right) \cdot \omega^2 r$$

mass in Δr is ... $\frac{\Delta r}{L} \times M$

$$\underbrace{T(r+\Delta r)}_{\text{Taylor Expansion}} = T(r) + \Delta r\left(\left.\frac{dT}{dr}\right|_r\right) + \text{neglect}$$

so, $$T(r) + \Delta r\left(\frac{dT}{dr}\right) - \cancel{T(r)} = -\frac{M}{L}\omega^2 r\,\Delta r$$

$$\frac{dT}{dr} = -\frac{M}{L}\omega^2 r$$

$$T(r) = T_0 - \frac{M}{2L}\omega^2 r^2$$

constant

when $r = L$, $T = 0$

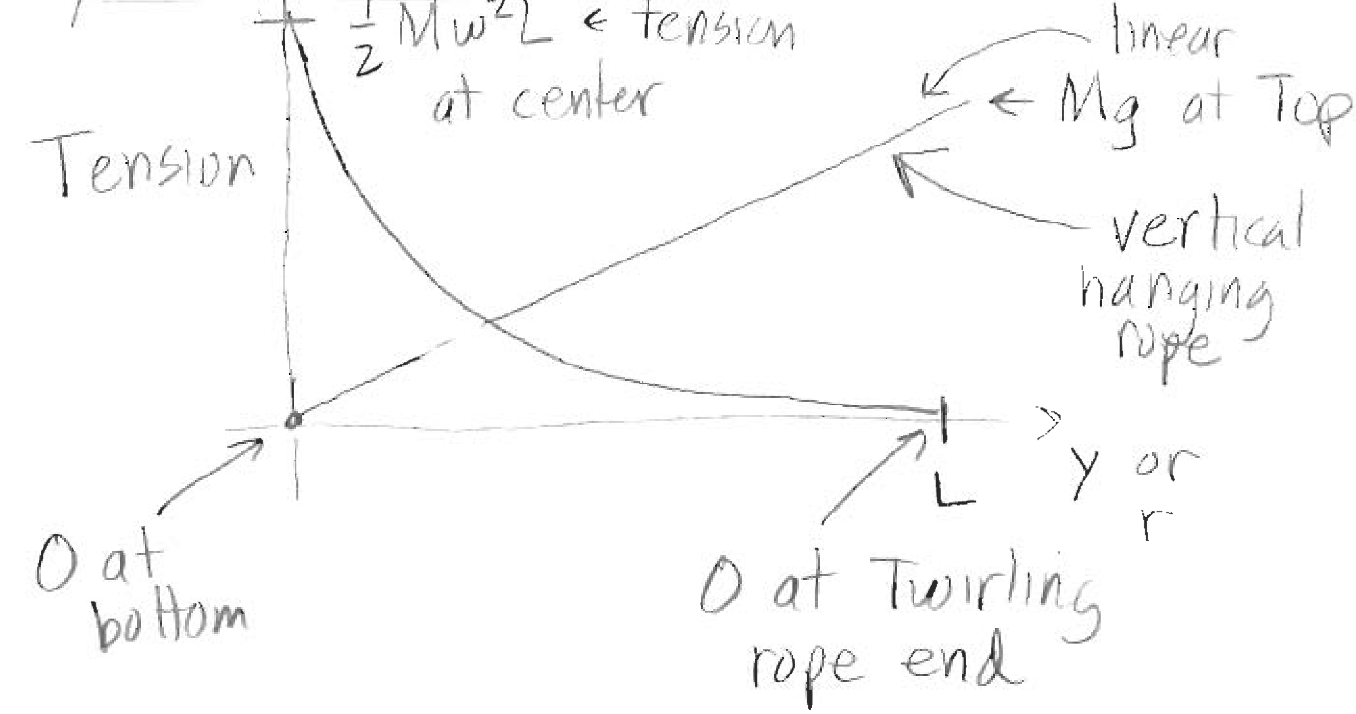
$$T(L) = T_0 - \frac{M}{2L}\omega^2 L^2 = 0$$

$$= T_0 - \frac{1}{2}M\omega^2 L = 0$$

$$T_0 = \frac{1}{2}M\omega^2 L$$

$$\boxed{T(r) = \frac{1}{2}\frac{M\omega^2}{L}\left(L^2 - r^2\right)}$$

$r = 0$, $= \frac{1}{2}M\omega^2 L$


$\frac{1}{2}M\omega^2 L \leftarrow$ tension at center
Tension
linear
$\leftarrow Mg$ at Top
vertical hanging rope
L
Y or r
0 at bottom
0 at Twirling rope end