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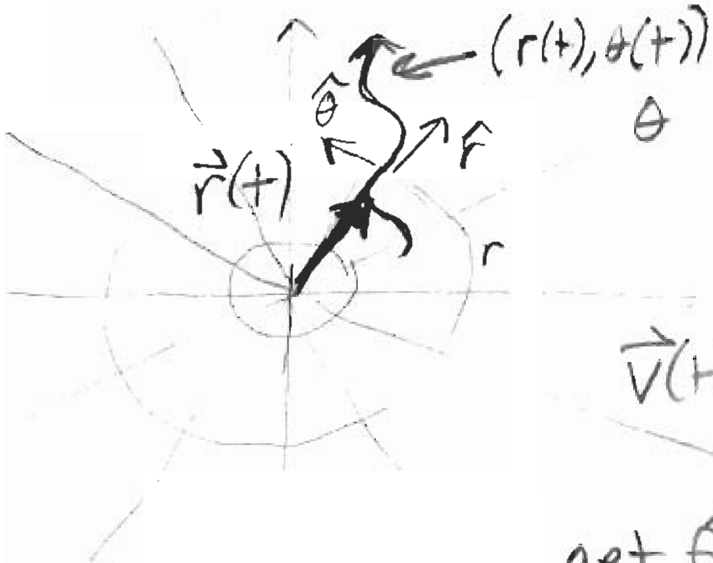
Working in Polar Coordinates



$\vec{r}(t) = r(t)\,\hat{r}$

$\hat{r}$ depends on θ!

$\vec{v}(t) = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$

$\dot{r}$, $r\dot{\theta}$: get from $r(t)$, $\theta(t)$

$\hat{r}$, $\hat{\theta}$: both depend on θ!

$$\vec{a}(t) = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

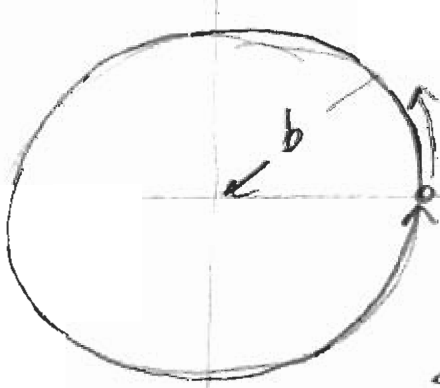
$\ddot{r}$: normal, radial acceleration

$r\dot{\theta}^2$: centripetal acceleration

$r\ddot{\theta}$: "normal" angular acceleration

$2\dot{r}\dot{\theta}$: hmm... "Coriolis"

Circular Motion



constant speed v around edge

time to go around ... T

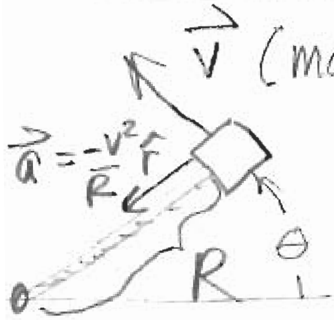
distance ... $2\pi b$

speed $= 2\pi \frac{b}{T} = v$

angular velocity: $\omega = \frac{2\pi}{T} = \frac{v}{b} = \dot{\theta}$

centripetal acceleration $= -b\omega^2 = -b \cdot \frac{v^2}{b^2} = -\frac{v^2}{b}$

Block on a string NO GRAVITY



$\vec v$ (magnitude v constant, direction changes, causing acceleration)

$r = R$ $\qquad \theta = \omega t$

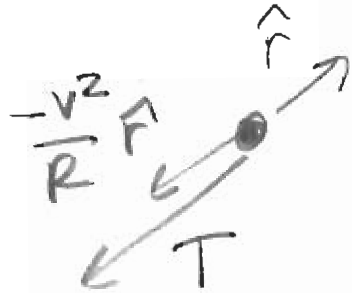
$\dot r = 0$ $\qquad = \frac{v}{R} t$

$\dot\theta = \frac{v}{R} = \omega$

$\ddot\theta = 0$

$$\vec a = (\underset{0}{\ddot r} - r\dot\theta^2)\hat r + (r\underset{0}{\ddot\theta} + 2\underset{0}{\dot r}\dot\theta)\hat\theta$$

$$\vec a = \left(-R\frac{v^2}{R^2}\right)\hat r = -\frac{v^2}{R}\hat r$$



Newton: $\vec F_{Net} = m\vec a$

$$-T = -m\frac{v^2}{R}$$

$$\boxed{T = \frac{mv^2}{R}}$$

Now add gravity!

It's not radial!!

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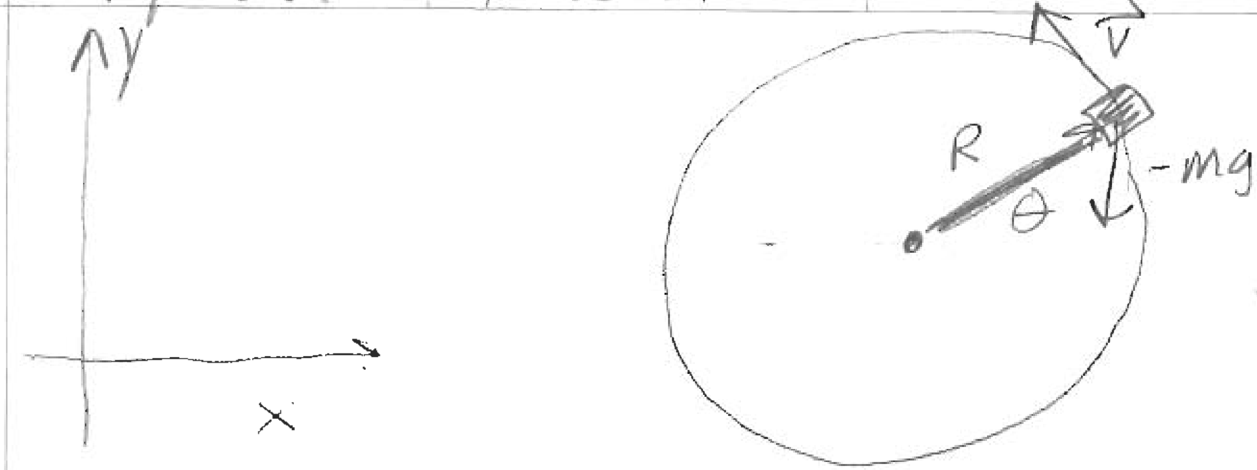


Same: $r(t) = R$, so $\dot{r} = 0$, $\ddot{r} = 0$
(circular motion)

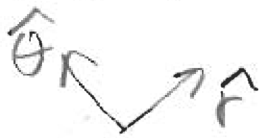
$\theta \to$ all bets are off!

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

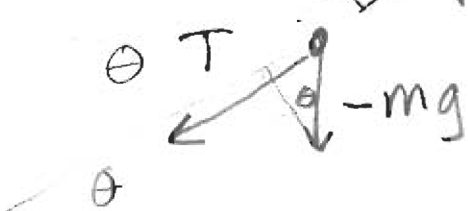
$$\vec{a} = -r\dot{\theta}^2\hat{r} + \underbrace{r\ddot{\theta}}_{\text{cannot neglect}}\hat{\theta}$$

← r info alone!

$$\vec{a} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$$ hmm...



$\boxed{\hat{r}}$ $-T - mg\sin\theta = -mR\dot{\theta}^2$



$\boxed{\hat{\theta}}$ $-mg\cos\theta = mR\ddot{\theta}$

not so easy to solve here...

Energy better

but: $T > 0$ (rope can't push)

$$T = mR\dot{\theta}^2 - mg\sin\theta > 0$$

$$R\dot{\theta}^2 > g\sin\theta$$

$$\frac{v^2}{R} > g\sin\theta$$

at top of arc, $\theta = 90^\circ = \pi/2$

$$\boxed{v > \sqrt{gR}} \quad \text{(demo)}.$$

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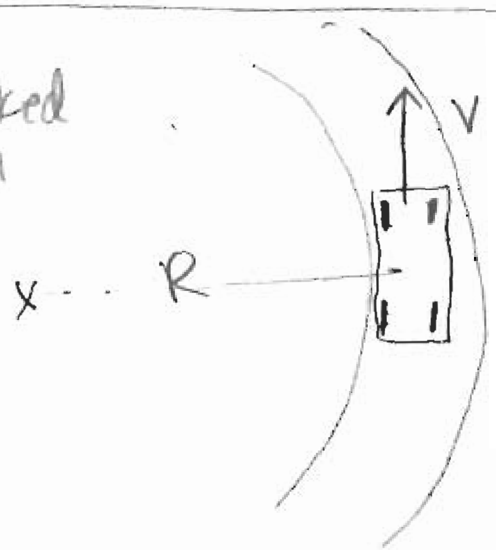
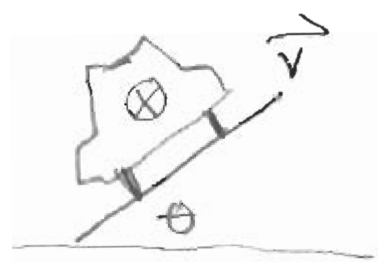
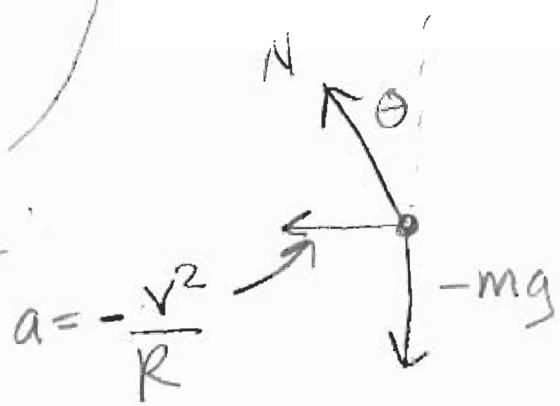
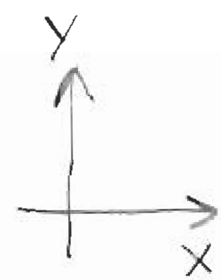
Car banked turn





Free Body:





Ideal Banking: Friction not needed!
$\vec{N}$ gives the acceleration

y: $\quad N\cos\theta - mg = m\ddot{y} = 0$

$$N = mg/\cos\theta$$

x: $\quad -N\sin\theta = -m\frac{v^2}{R} = -mg\frac{\sin\theta}{\cos\theta} = m\ddot{x}$