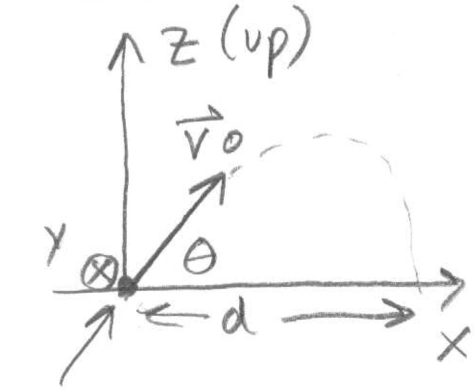
Back to trajectories, (text page 19)




t = 0
mass m

How far does the mass go before return to the ground!

Think: $\theta = 0$ ...
distance = 0
$\theta = 90°$, same!

Key concept: independence of motion in x + z (horizontal + vertical).

easy: horizontal:

$$x = x_0 + v_{0x} t$$

since start at origin $x_0 = 0$

$v_{0x} = v_0 \cos\theta$

vertical:

$$z = z_0 + v_{0z} t - \frac{1}{2} g t^2$$

$z_0 = 0$

$v_0 \sin\theta$

gravity

so:

$$\boxed{\begin{aligned} x &= v_0 \sin\theta\, t \\ z &= v_0 \cos\theta\, t - \frac{1}{2} g t^2 \end{aligned}}$$

Eliminate $t$ in favor of $x$ ...

$$t = \frac{x}{v_0 \sin\theta}$$

then,
$$z = \frac{v_0 \cos\theta}{v_0 \sin\theta}\, x - \frac{1}{2} g \frac{x^2}{v_0^2 \sin^2\theta}$$

$$z = \frac{x}{\tan\theta} - \frac{1}{2}\frac{g}{v_0^2 \sin^2\theta}\, x^2$$

ground: $z = 0 = \underbrace{x}_{0}\left(\frac{1}{\tan\theta} - \frac{1}{2}\frac{g}{v_0^2\sin^2\theta}\, x\right)$

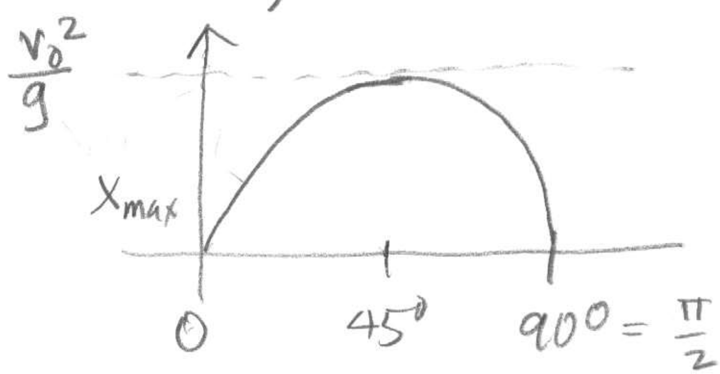
$$\frac{1}{\tan\theta} = \frac{1}{2}\frac{g}{v_0^2\sin^2\theta}\, x$$

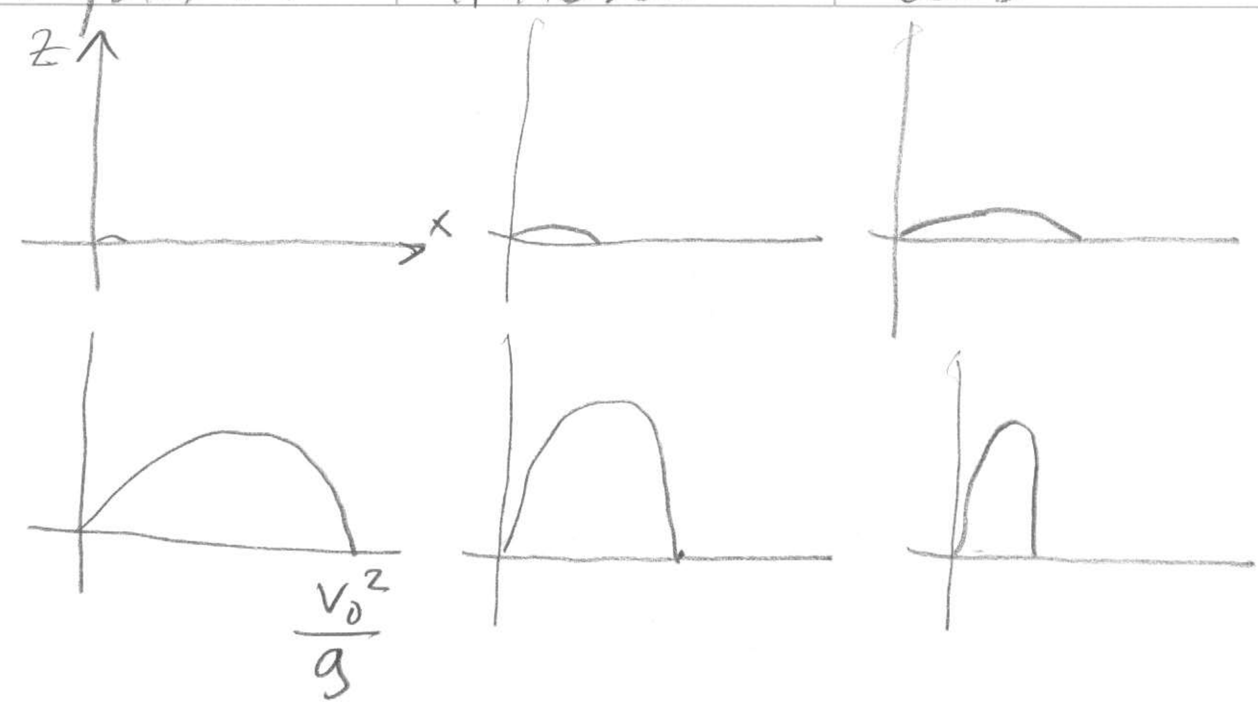
$$x = \frac{2 v_0^2 \sin^2\theta}{g} \cdot \frac{\cos\theta}{\sin\theta}$$

$$x = 2\frac{v_0^2}{g} \sin\theta\cos\theta$$

$\boxed{x = 0}$ $\qquad$ $\boxed{x_{max} = \frac{v_0^2}{g}\sin(2\theta)}$

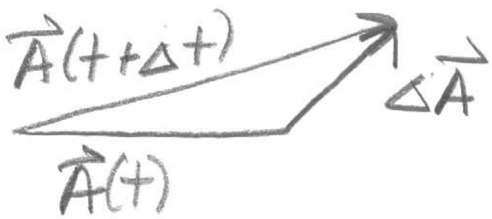
$$\theta = 0, 90^0 \rightarrow \sin(2\theta) = 0$$






## Derivative of a vector (p. 23)

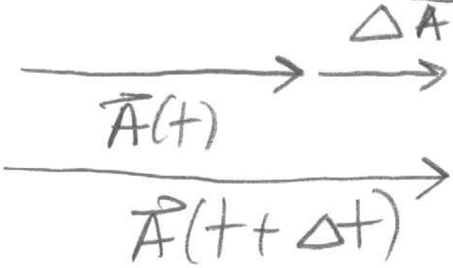
$$\Delta\vec{A} = \vec{A}(t+\Delta t) - \vec{A}(t)$$




$$\frac{d\vec{A}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{A}}{\Delta t}$$

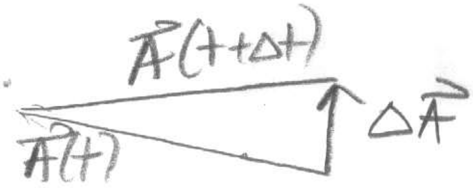
$\Delta\vec{A}$ has two ways to arise...

(1) $\vec{A}(t)$ can change in length




(2) $\vec{A}(t)$ can change in direction

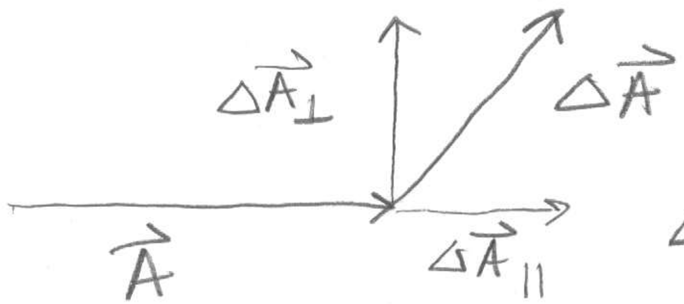
important for circular motion!




as $\Delta t \to 0$

$\Delta\vec{A} \perp$ to $\vec{A}$

General Case ...

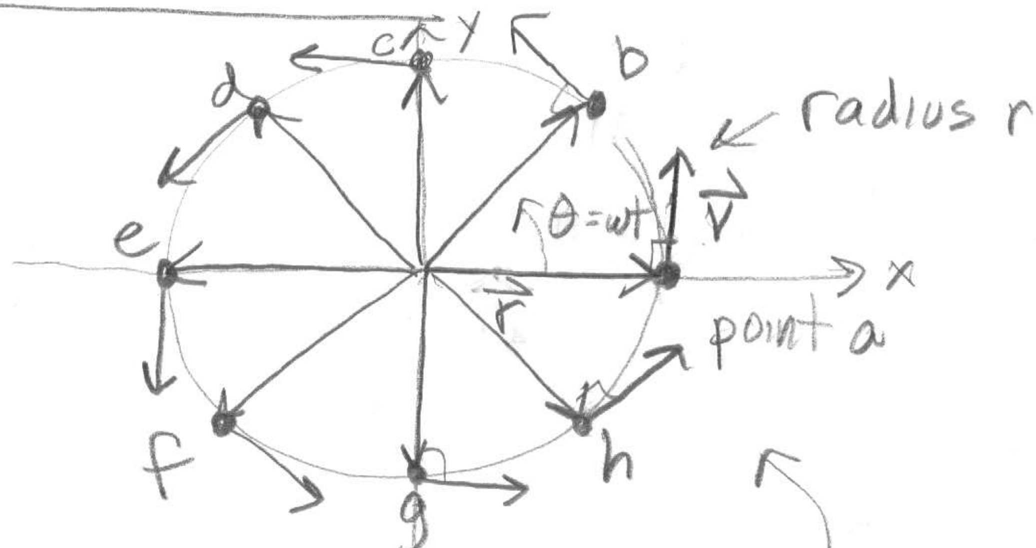



remember ...

$$\Delta\vec{A}_{\parallel} = (\Delta\vec{A}\cdot\hat{A})\,\hat{A}$$

$$\Delta\vec{A}_{\perp} = (\hat{A}\times\Delta\vec{A})\times\hat{A}$$

## Circular Motion

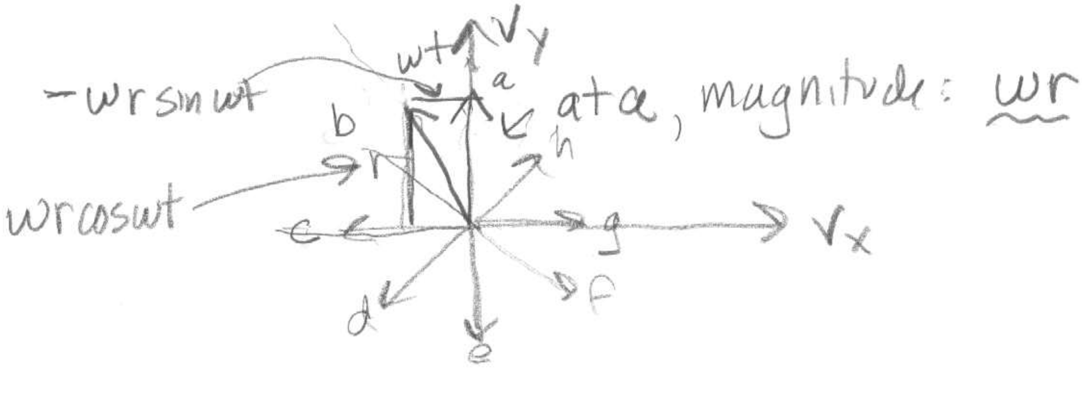



$$\vec{r} = r(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath})$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \underset{\text{constant}}{r}\,(-\omega\sin\omega t\,\hat{\imath} + \omega\cos\omega t\,\hat{\jmath})$$

$$\vec{r}\cdot\vec{v} = r^2(-\omega\sin\omega t\cos\omega t + \omega\sin\omega t\cos\omega t) = 0$$

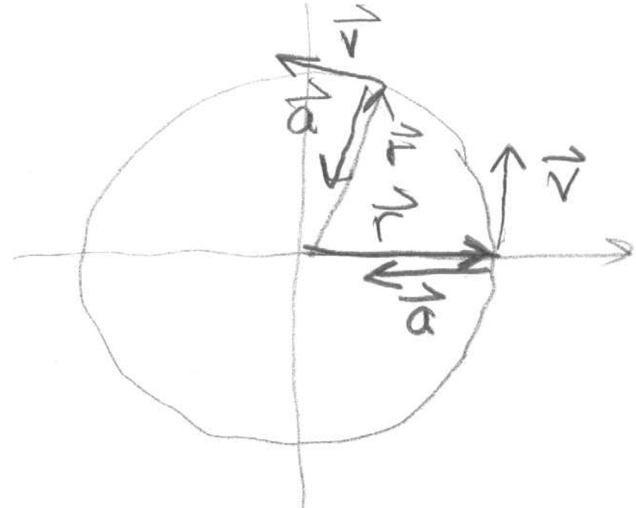
means $\vec{r} \perp$ to $\vec{v}$



$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = r\omega\left(-\omega\cos(\omega t)\,\hat{\imath} - \omega\sin(\omega t)\,\hat{\jmath}\right)$$

$$= -\omega^2 r\left(\cos(\omega t)\,\hat{\imath} + \sin(\omega t)\,\hat{\jmath}\right)$$

$$\ddot{\vec{r}} = -\omega^2\vec{r}$$



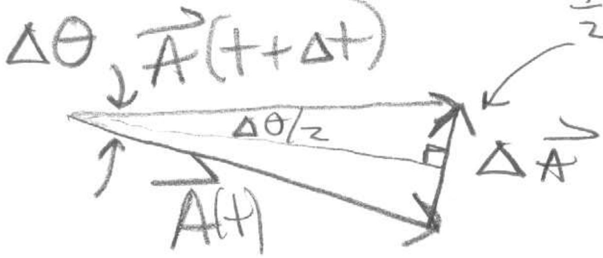

centripetal acceleration

$|\vec{r}| = r$

$|\vec{v}| = \omega r$

$|\vec{a}| = \omega^2 r$

another way to look at this..




$\frac{1}{2}|\Delta\vec{A}| = |\vec{A}| \underbrace{\sin\frac{\Delta\theta}{2}}$

$\Delta\theta$ small, $\simeq \frac{\Delta\theta}{2}$

$|\vec{A}(t+\Delta t)| = |\vec{A}(t)|$

$$|\Delta\vec{A}| = |\vec{A}|\,\Delta\theta$$

$$\left|\frac{\Delta\vec{A}}{\Delta t}\right| = |\vec{A}|\,\frac{\Delta\theta}{\Delta t}$$

$$\left|\frac{d\vec{A}}{dt}\right| = |\vec{A}|\,\frac{d\theta}{dt}$$

only when $|\vec{A}| = \text{constant}$

$\vec{r}$ before, $\theta = \omega t$

$$\frac{d\theta}{dt} = \omega$$

$$|\vec{v}| = \left|\frac{d\vec{r}}{dt}\right| = |\vec{r}|\,\omega = \omega r$$