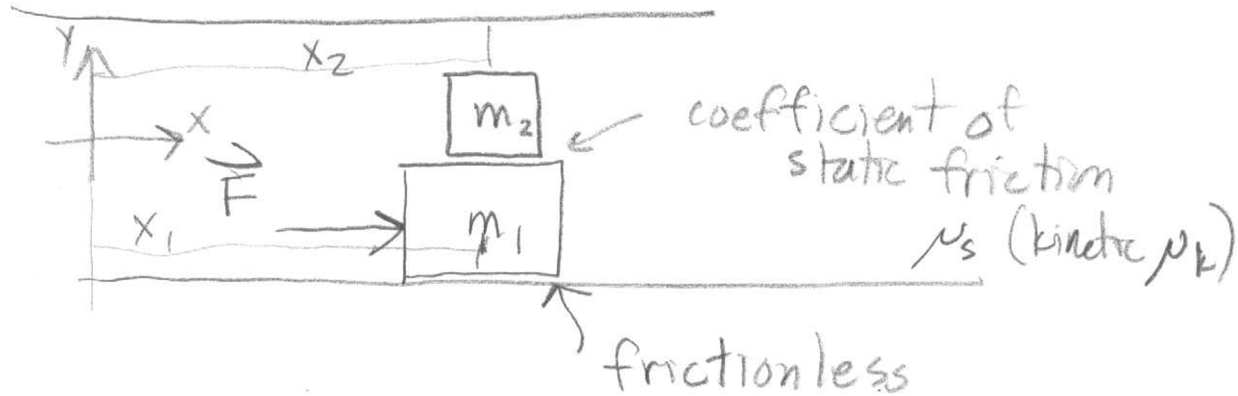
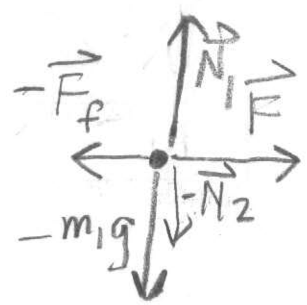
Accelerated Frame



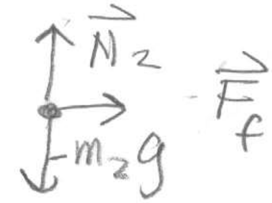
1) what is the largest $\vec{F}$ that does not make the block m_2 slip?
2) What is the acceleration of m_1, as a function of F? (Just after applying $\vec{F}$)
3) Analyze m_2 in the accelerated frame

1)

m_1



m_2



y: $N_1 - N_2 - m_1 g = 0$ $\qquad$ $N_2 - m_2 g = 0$

x: $m_1 \ddot{x}_1 = F - F_f$ $\qquad$ $m_2 \ddot{x}_2 = F_f$

constraint $\quad x_1 = x_2 \rightarrow \ddot{x}_1 = \ddot{x}_2$

$$\ddot{x}_2 = \frac{F_f}{m_2} = \ddot{x}_1$$

$$\frac{m_1}{m_2} F_f = F - F_f$$

$$\left(1 + \frac{m_1}{m_2}\right) F_f = F \qquad \underline{F_f = \frac{m_2}{m_1 + m_2} F}$$

but, $F_f \leq \mu_s N_2 = \mu_s m_2 g$

so $$\frac{m_2}{m_1+m_2} F < \mu_s m_2 g$$

$$\boxed{F < \mu_s (m_1 + m_2) g}$$

maximum

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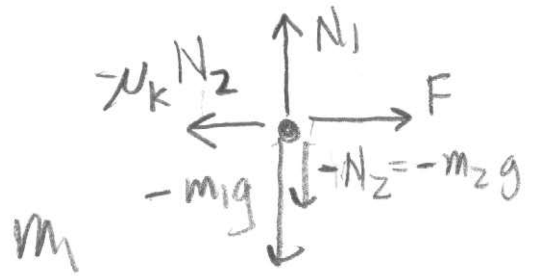
2) Prior to slip: $\ddot{x}_2 = \ddot{x}_1 = \frac{F_f}{m_2} = \frac{F}{m_1+m_2}$

this was obvious, actually ...

$$\underbrace{(m_1+m_2)}_{\text{total mass}} \begin{pmatrix} \ddot{x}_1 \\ \shortparallel \\ \ddot{x}_2 \end{pmatrix} = F$$

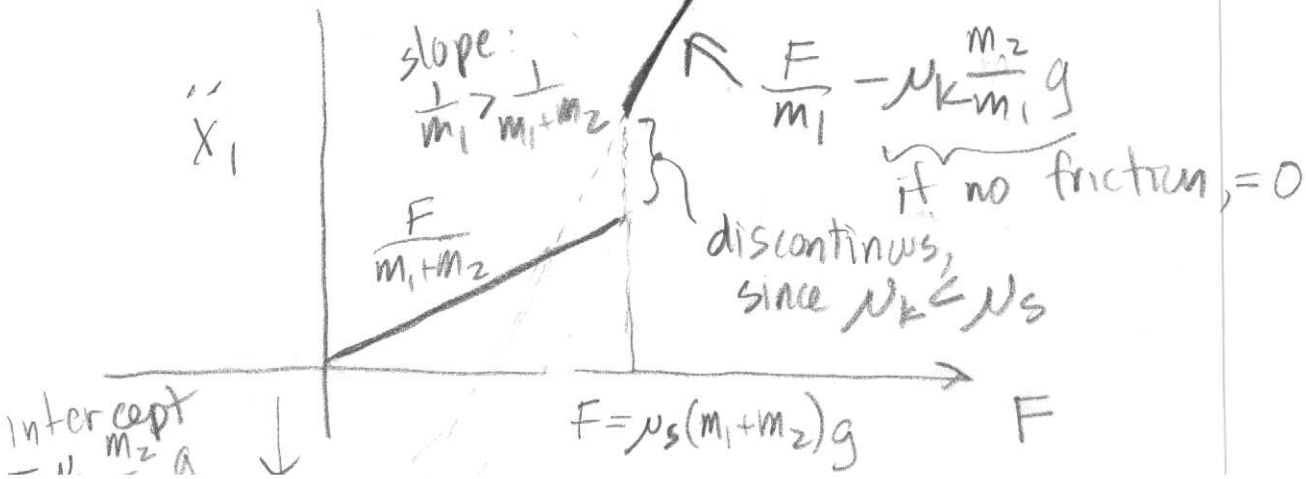
when slipping starts ...



$$m_1 \ddot{x}_1 = F - \mu_k N_2$$

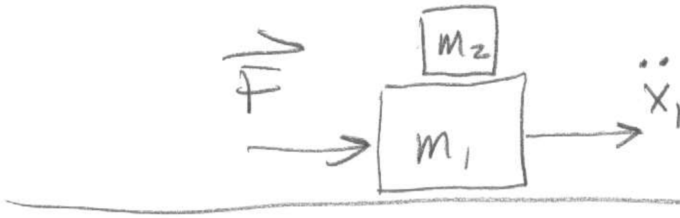
$$m_1 \ddot{x}_1 = F - \underbrace{\mu_k m_2 g}_{\text{smaller in mag. than } \mu_s m_2 g}$$

$$\ddot{x}_1 = \frac{F}{m_1} - \mu_k \frac{m_2}{m_1} g$$



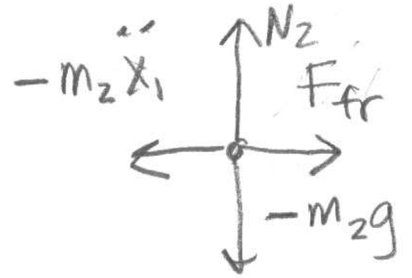
3) $\vec{F}_{apparent} = \vec{F}_{true} - M\ddot{\vec{R}}$

describing accelerating frame.



(look in that frame)

before slip: $\ddot{x}_1 = \dfrac{F}{m_1 + m_2}$



m_2 at rest in this frame.

gives maximum acc

$$F_{fr} - m_2\ddot{x}_1 = 0$$

$$F_{fr} = m_2\ddot{x}_1 = \frac{m_2}{m_1+m_2}F$$

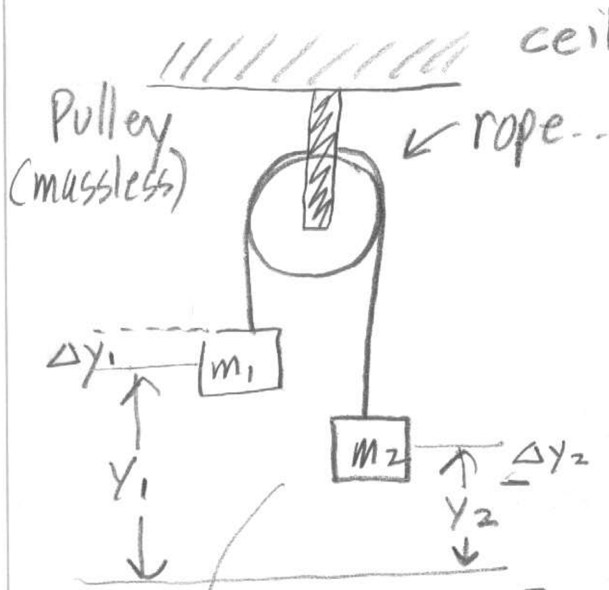
(see earlier)

$$F_{fr} < \mu_s N_2 = \mu_s m_2 g$$

$$\frac{\cancel{m_2}}{m_1+m_2}F < \mu_s \cancel{m_2} g$$

$$F < \mu_s (m_1+m_2) g.$$

Atwood's Machine



First: imagine applying

$m_1 \xrightarrow{T_1}$

$m_1 \ddot{y}_1 = T_1$

string: $T_1 \leftarrow \bullet \rightarrow T_2$

$\underbrace{m_s \ddot{y}_s = T_2 - T_1}$

but $m_s = 0$

$\therefore T_2 = T_1$

m_2: $T_2 \leftarrow \bullet \rightarrow F$

$m_2 \ddot{y}_2 = F - T_2$

now...

m_1: T up, $-m_1 g$ down

m_2: T up, $-m_2 g$ down

$$m_1 \ddot{y}_1 = T - m_1 g \qquad m_2 \ddot{y}_2 = T - m_2 g$$

3 unknowns...

$$\Delta y_1 = -\Delta y_2$$

$$\ddot{y}_1 = -\ddot{y}_2$$

$$-\ddot{y}_1 = \frac{T}{m_2} - g$$

$$m_1\left[-\frac{T}{m_2} + g\right] = T - m_1 g$$

$$2 m_1 g = T\left(1 + \frac{m_1}{m_2}\right)$$

$$\boxed{T = \frac{2 m_1}{1 + \frac{m_1}{m_2}} g = 2 \frac{m_1 m_2}{m_1 + m_2} g}$$

$$\ddot{y}_1 = -\frac{T}{m_2} + g = -2\frac{m_1 m_2}{m_2(m_1+m_2)} g + g$$

$$= \frac{-2m_1m_2 + m_2m_1 + m_2^2}{m_2(m_1+m_2)} g$$

$$= \frac{m_2(m_2-m_1)}{m_2(m_1+m_2)} g$$

$$\boxed{\begin{aligned} \ddot{y}_1 &= \left(\frac{m_2-m_1}{m_1+m_2}\right) g \\ \ddot{y}_2 = -\ddot{y}_1 &= \left(\frac{m_1-m_2}{m_1+m_2}\right) g \end{aligned}}$$

$\leftarrow m_2 > m_1,\ \ddot{y}_1 > 0$

$\ddot{y}_2 < 0$

$$T = 2\frac{1}{\frac{1}{m_1 m_2}(m_1+m_2)} g$$

$$= 2\left(\frac{1}{\frac{1}{m_1}+\frac{1}{m_2}}\right) g \qquad = 2\mu g$$

Reduced mass . . . $\frac{1}{\mu} \equiv \frac{1}{m_1} + \frac{1}{m_2}$

$$\mu = \frac{1}{1/m_1 + 1/m_2}$$

The smaller mass dominates

$m_1 = 1$ kg $\qquad m_2 = 3$ kg.

$$\boxed{\mu = \frac{1}{\frac{1}{1}+\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}\text{ kg}}$$