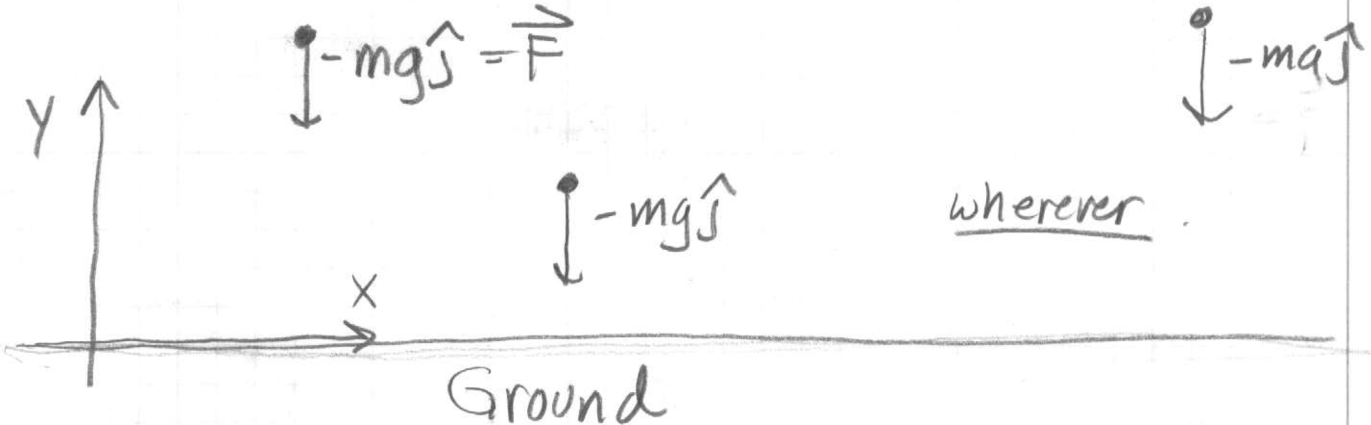
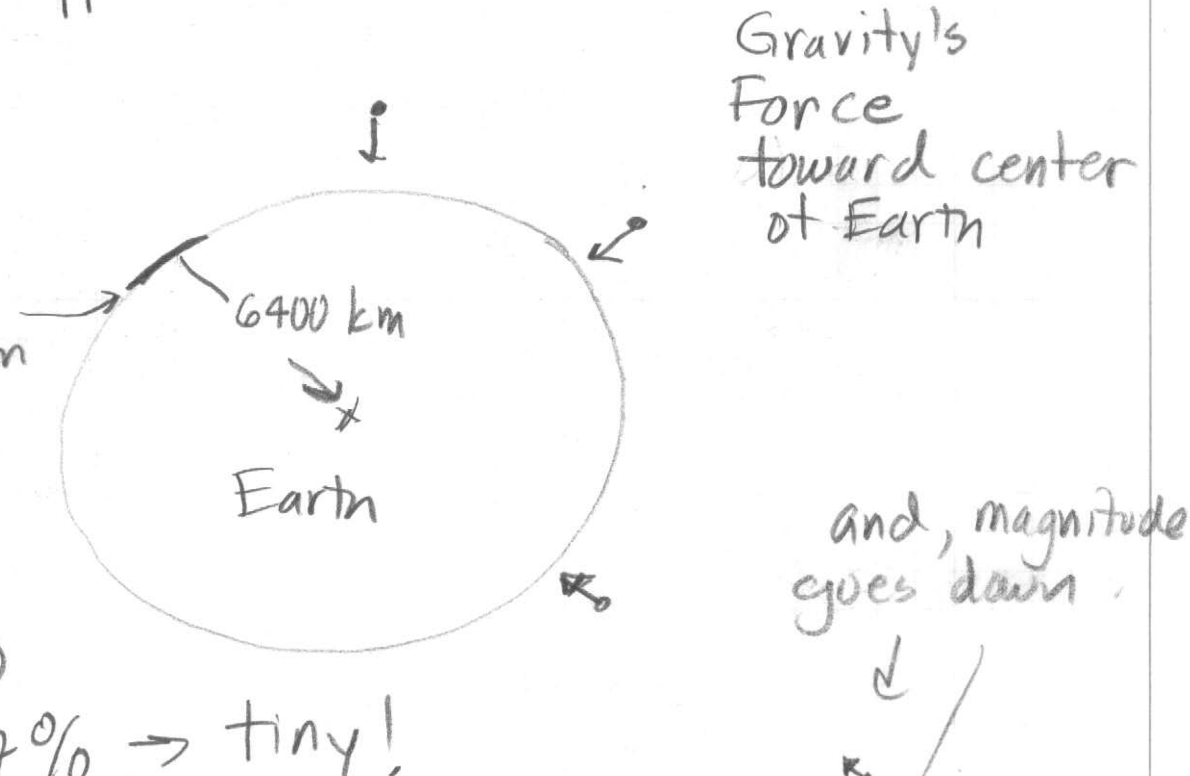
The most famous force of all... gravity.

Near Earth, a point mass m will feel a downward force that is a constant vector downward...



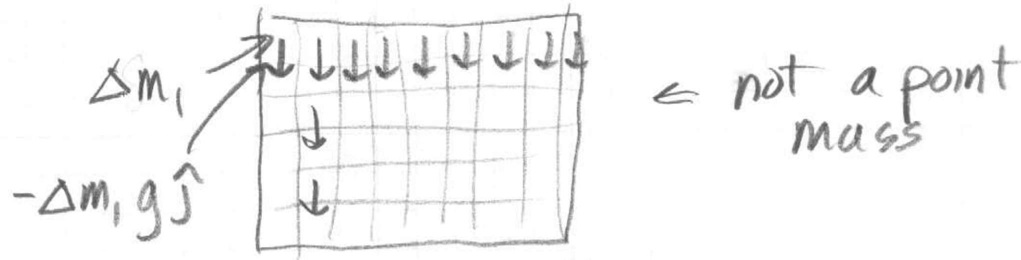
Actually, constant gravity near earth is an approximation...



$\frac{10}{6400} \simeq \frac{1}{640}$

$\sim 0.17\% \rightarrow$ tiny!

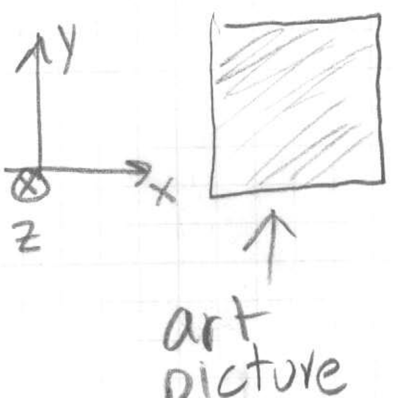
over distance we live, this change is negligible



$$\text{Total Force} = -\sum_{i=1}^{N} \Delta m_i g \hat{j}$$

$$= -Mg\hat{j}$$

→ "Where" → doesn't matter yet.



$$-Mg\hat{j} = M\vec{a} = M(a_x\hat{i} + a_y\hat{j} + a_y\hat{k})$$

"free body" diagram
put forces acting on one body
→ tails on dot..

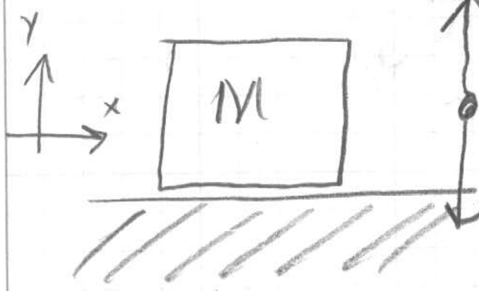
Falling mass: $Ma_y = M\dfrac{d^2y}{dt^2} = -Mg$

$\dfrac{d^2x}{dt^2} = 0$ $\qquad \dfrac{d^2y}{dt^2} = -g$ $\qquad \dfrac{d^2z}{dt^2} = 0$

$\dfrac{dx}{dt} = v_{x0}$ $\qquad \dfrac{dy}{dt} = v_{y0} - gt$ $\qquad \dfrac{dz}{dt} = v_{z0}$

$$\boxed{x = x_0 + v_{x0}t \qquad y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \qquad z = z_0 + v_{z0}t}$$

more later.



$\vec{N} = N\hat{j}$ ($\perp$ to surface).

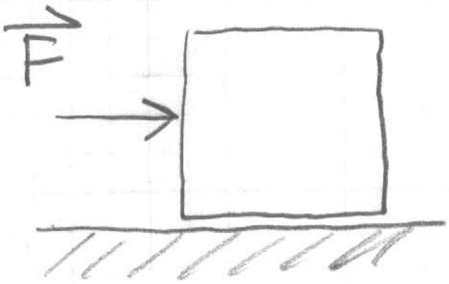
$-Mg\hat{j}$ ← but $\vec{a} = 0$!

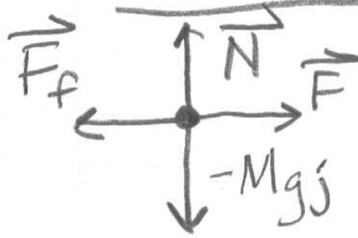
$$M\vec{a} = M \cdot 0 = N\hat{j} - Mg\hat{j} = 0$$

$$\boxed{N = Mg}$$

Normal force is electrostatic in origin.

$\boxed{\text{Friction}}$ resists forces ∥ to surface.





$\vec{F}_f$: direction ∥ to surface. opposes other ∥ forces.

$$|\vec{F}_f| \leq \mu_s |\vec{N}| \quad \mu_s \rightarrow \text{no dimensions}$$

so, as long as the applied force is less in magnitude than ...

vertical: $N - Mg = 0$

$$N = Mg$$

$|\vec{F}_f| \leq \mu_s Mg$ ← block doesn't move

When the applied force exceeds $\mu_s Mg$, the block starts to move, and the force of friction goes down!!

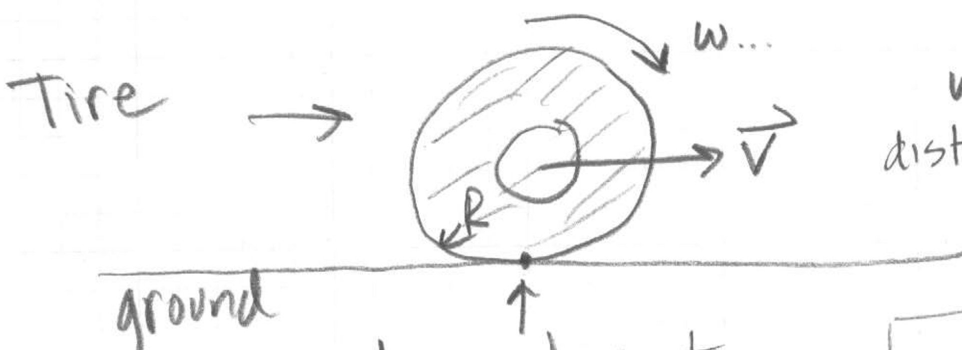
Still $\propto |\vec{N}|$, but now

SLIDING: $|\vec{F}_f| = \mu_k |\vec{N}|$

$\mu_s \Rightarrow$ called "coefficient of static friction"

$\mu_k \Rightarrow$ called "coefficient of kinetic friction"

usually $\mu_k < \mu_s$ .. why you don't want to skid in a car..



$\omega T = 2\pi$

distance: $vT = 2\pi R$

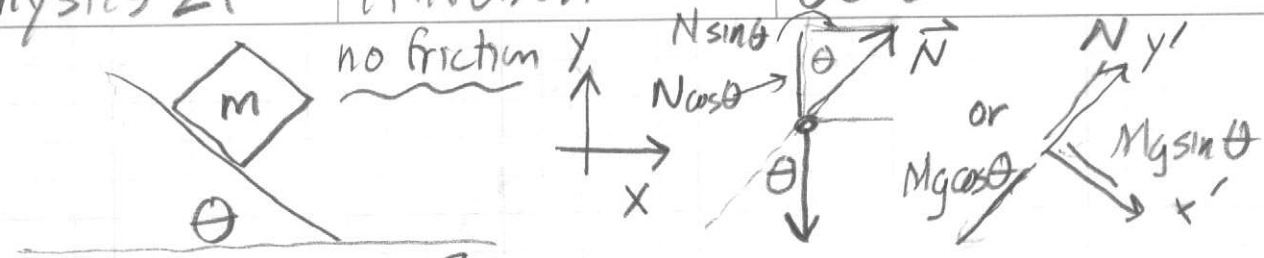
$T = \frac{2\pi R}{v}$

$\omega \cdot T = \frac{\omega \cdot 2\pi R}{v} = 2\pi$

$\boxed{\omega = \frac{v}{R}}$

tire at rest w/r to ground here

$\rightarrow \mu_s$ matters!



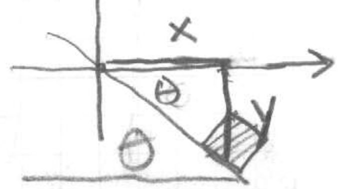
$\frac{dx}{dt} \equiv \dot{x} \quad \frac{d^2x}{dt^2} = \ddot{x}$

x :

$N \sin\theta = M\ddot{x}$

y :

$N\cos\theta - Mg = M\ddot{y}$ (3 unknowns)



constraint

$\tan\theta = \frac{-y}{x}$ 3rd equation

$x\tan\theta = -y$

$\ddot{x}\tan\theta = -\ddot{y}$

$N\cos\theta - Mg = -M\ddot{x}\tan\theta$

$= -N\sin\theta\tan\theta$

$N\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) = Mg$

$N\left(\frac{\overbrace{\cos^2\theta + \sin^2\theta}^{1}}{\cos\theta}\right) = Mg$

$N = Mg\cos\theta$

$m\ddot{x} = Mg\cos\theta\sin\theta$

$m\ddot{y} = Mg(\cos^2\theta - 1) = -Mg\sin^2\theta$

x' :

$M\ddot{x}' = Mg\sin\theta$

$M\ddot{y}' = 0$ (sliding along incline)

$= N - Mg\cos\theta$

$N = Mg\cos\theta$

$\ddot{x} = \ddot{x}'\cos\theta$

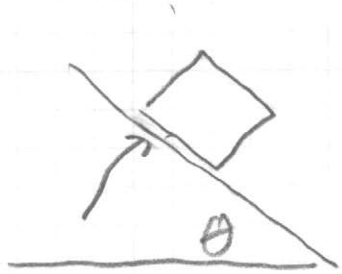
$= Mg\cos\theta\sin\theta$

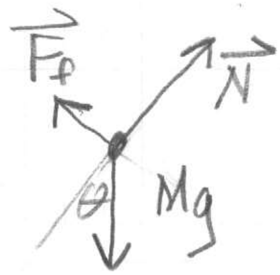
$\ddot{y} = -\ddot{x}'\sin\theta$

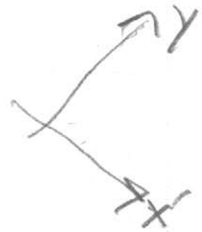
$\ddot{y} = -Mg\sin^2\theta$

Now add friction... How large can θ get before block slides?

use tilted coordinate system







y':
$$N - Mg\cos\theta = 0$$
$$N = Mg\cos\theta$$
$$|\vec{F_f}| \leq \mu_s N = \mu_s Mg\cos\theta$$

x':
$$Mg\sin\theta - F_f = m\ddot{x}'$$

as long as $Mg\sin\theta < \mu_s Mg\cos\theta$ block won't move

$$\tan\theta < \mu_s$$

$$\theta_{max} = \tan^{-1}(\mu_s)$$