Can continue...

\[ f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \frac{1}{3!} f'''(a)(x-a)^3 + \ldots \]

\[
\quad + \frac{1}{4 \cdot 3!} f^{(4)}(a) (x-a)^4 + \ldots \]

\[
\quad + \frac{1}{4!} \quad \text{"Taylor Series"}
\]

\[ f(x) = \sin(x), \quad a = 0 \]

\[
\begin{align*}
  f'(x) &= \cos(x) \\
  f''(x) &= -\sin(x) \\
  f'''(x) &= -\cos(x) \\
  f^{(4)}(x) &= \sin(x) \\
  f^{(5)}(x) &= \cos(x)
\end{align*}
\]

\[
\begin{align*}
  f(a) &= \sin(0) = 0 & f'(a) &= \cos(0) = 1 \\
  f''(a) &= -\sin(0) = 0 & f''(a) &= -\cos(0) = -1 \\
  f'''(a) &= \sin(0) = 0 & f^{(4)}(a) &= \cos(0) = 1
\end{align*}
\]

\[
\sin(x) = 0 + \frac{1}{1!} \cdot 1 \cdot x + \frac{1}{2!} \cdot 0 \cdot x^2 + \frac{1}{3!} \cdot (-1) x^3 + \frac{1}{4!} \cdot 0 \cdot x^4 + \frac{1}{5!} \cdot 1 \cdot x^5
\]

around \( x = 0 \)

\[
\sin(x) \approx x - \frac{1}{6} x^3 + \frac{1}{120} x^5
\]

around \( x = 0 \)
\[
\cos(x) = 1 - \frac{1}{2} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \ldots
\]

around \( x = 0 \)

Another famous series

\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots
\]

\[
= (1-x) \left( 1 + x + x^2 + x^3 + x^4 + \ldots \right)
\]

\[
= 1 + x - x + x^2 - x^2 + \ldots
\]

\[
= 1 \checkmark
\]

as a Taylor Series

\[
f(x) = \frac{1}{1-x}
\]

choose \( a = 0 \)

\[
f'(0) = 1
\]

\[
f'(x) = \frac{-1}{(1-x)^2} \left( -1 \right)
\]

\[
= \frac{1}{(1-x)^2}
\]

\[
f''(0) = 1
\]

\[
f''(x) = -\frac{2}{(1-x)^3} \left( -1 \right) = \frac{2}{(1-x)^3}
\]

\[
f''(0) = 2
\]

\[
f'''(x) = -\frac{3 \cdot 2}{(1-x)^4} \left( -1 \right) = \frac{3!}{(1-x)^4}
\]

\[
f'''(0) = 3!
\]

\[
f^{(n)}(x) = \frac{1}{(1-x)^n}
\]

\[
f^{(n)}(0) = \frac{1}{n!}
\]
\[ \tan(x) \approx \arctan(0) \]
\[ \tan(0) = 0 \]

\[ f'(x) = \left( \frac{\sin(x)}{\cos(x)} \right)' = \frac{\cos(x) \cdot \cos^2(x) - \sin(x) \cdot (-\sin(x))}{\cos^2(x)} \]
\[ = 1 + \frac{\sin^2(x)}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = 1 \]

\[ f''(x) = \frac{1}{\cos^2(x)} \]
\[ f''(0) = 1 \]

\[ f'''(x) = \frac{2 \sin(x)}{\cos^3(x)} \]
\[ f'''(0) = 0 \]

\[ f''''(x) = 2 \left( \frac{\cos(x)}{\cos^3(x)} - \frac{3 \sin(x)}{\cos^4(x)} \right) \]
\[ = 2 \left( \frac{1}{\cos^2(x)} + \frac{3 \sin^2(x)}{\cos^4(x)} \right) \]
\[ f''''(0) = 2 \left( \frac{1}{1} + \frac{3 \cdot 0}{1} \right) = 2 \]

\[ \tan(x) \approx 0 + x + 0 + \frac{2}{3!} x^3 \]
\[ \tan(x) \approx x + \frac{1}{3} x^3 \left[ + \frac{2}{15} x^5 + \frac{17}{315} x^7 \right] \]
Newton's Laws

First Law: In absence of (N1) outside influences (Forces), the velocity of a body as viewed from an "inertial coordinate system" will stay constant.

Comments: "Inertial System"
- no friction (outer space)
- let go of an object... if no forces on you or it, it will stay at rest.
- Then, study other bodies...
- \[ \frac{d\vec{V}_{\text{body}}}{dt} = 0 \] as long as no messing with the body

Second Law (N2)

\[ \vec{F}_{\text{net}} = m\vec{a} \]

identify all forces that act on a body. This can get subtle!
\[ \mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_N \]

Tip to tail, components. \[ \sum_{i=1}^{N} \mathbf{F}_i \]

\[ m = \text{mass} \quad \text{(related to weight, but same on all)} \]

\[ \text{(measurable) planets...} \]

\[ \mathbf{a} = \text{the one and only} \]

\[ \text{acceleration of the body, as observed from an inertial frame (look out when viewing from a non-inertial!)} \]

\[ \mathbf{F} \Rightarrow \text{abstraction!} \]

\[ \mathbf{a} \Rightarrow \text{measurable, most frequently} \quad \mathbf{a} = 0 \]

\[ \text{(Statics!)} \]

\[ \mathbf{a} = 0, \text{ does that mean no forces act on an object?} \]

\[ \text{NOPE, no net force.} \]
Third Law (N3)

If body b pushes body a with force $F_a$, then a pushes back on b with force $-F_a$.

→ LOOK OUT... 2 bodies, 2 forces, one force for each body

**Box on Table (at rest)**

<table>
<thead>
<tr>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
</tbody>
</table>

No forces

$a_x = 0$

Weight on earth = $-m \cdot g \cdot \hat{y}$

\[ g = "acceleration \ of \ gravity" = 9.8 \ \frac{m}{s^2} \]

Not quite the same everywhere, but down toward earth's center everywhere.

Moon $\rightarrow$ not $g$, etc.

Is that all? $\vec{a} = 0$, so net force 0 too!