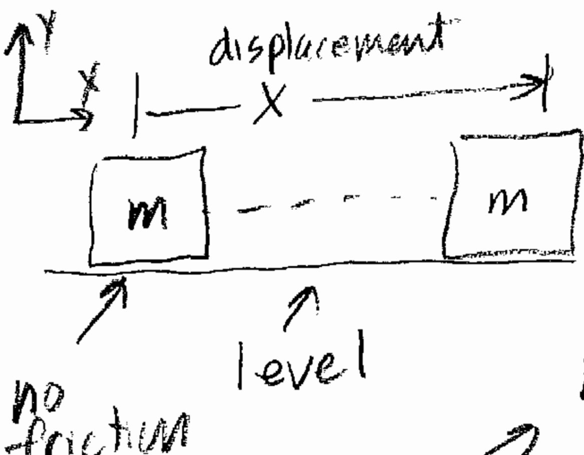
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

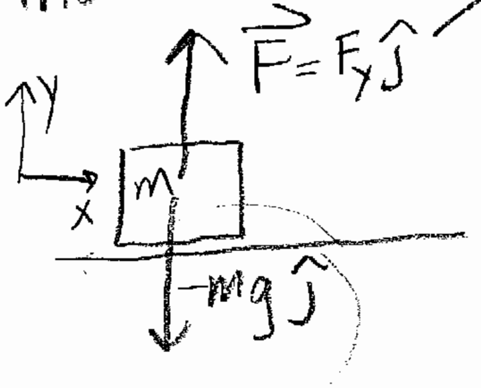
Some physical examples

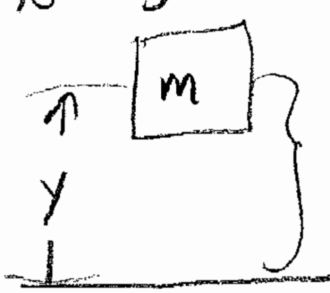
Work ... displacement × component of force ∥ to displacement.



no $\vec{F}$, no work! (done by you)

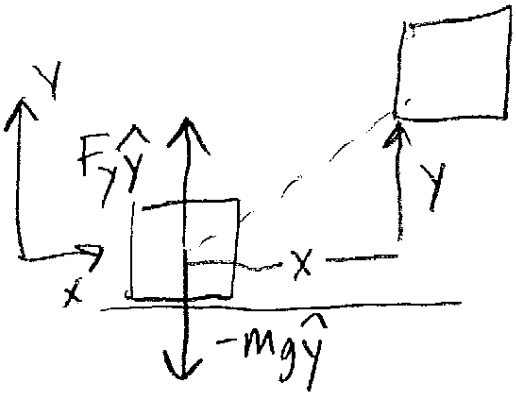
$F_y\hat{\jmath} - mg\hat{\jmath} = 0$, $F_y = mg$





Work $= F_y\, y$

nobody does gravities work



displacement

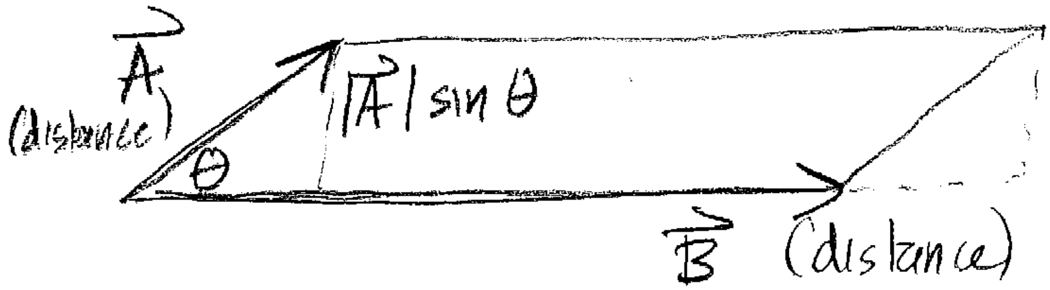
$$\vec{d} = x\hat{\imath} + y\hat{\jmath}$$

$$\text{Work} = \vec{F}\cdot\vec{d} = F_y\hat{\jmath}\cdot(x\hat{\imath}+y\hat{\jmath})$$

0

$$\vec{F}\cdot\vec{d} = F_y\, y$$

<u>Cross Product</u>: area of a parallelogram



$$\text{area} = |\vec{A}|\sin\theta\cdot|\vec{B}|$$

$$= |\vec{A}\times\vec{B}|$$

Po

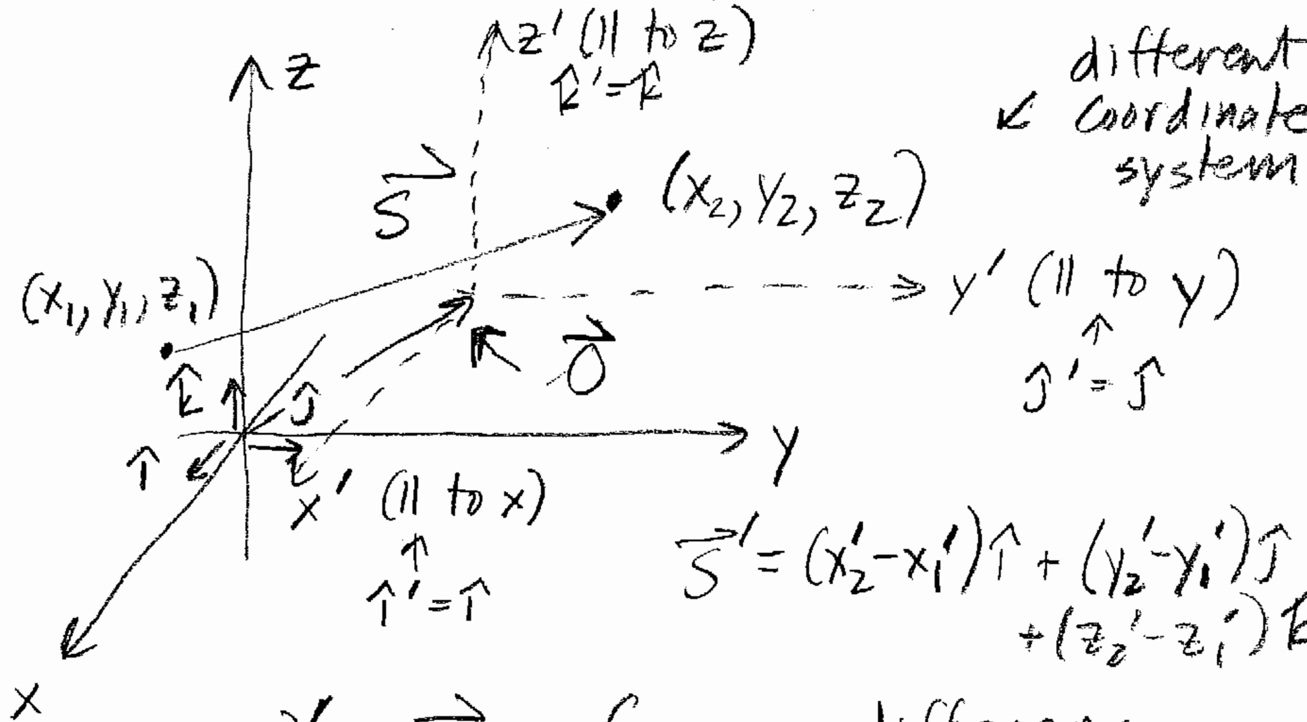
Displacement Vector (K+K section 1.5 pp. 11-13)

start at (x_1, y_1, z_1)
end at (x_2, y_2, z_2)

measure in meters unless told otherwise!

Displacement Vector

$$\vec{S} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$



$$\vec{S}' = (x_2' - x_1')\hat{\imath} + (y_2' - y_1')\hat{\jmath} + (z_2' - z_1')\hat{k}$$

$\vec{S}' = \vec{S}$ (since differences are involved)

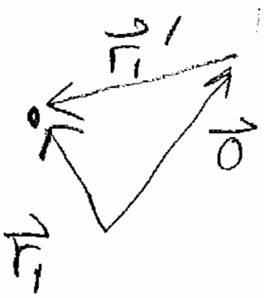
---

but...

"position vector"

$$\vec{r}_1 = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$$

does depend on coordinates!



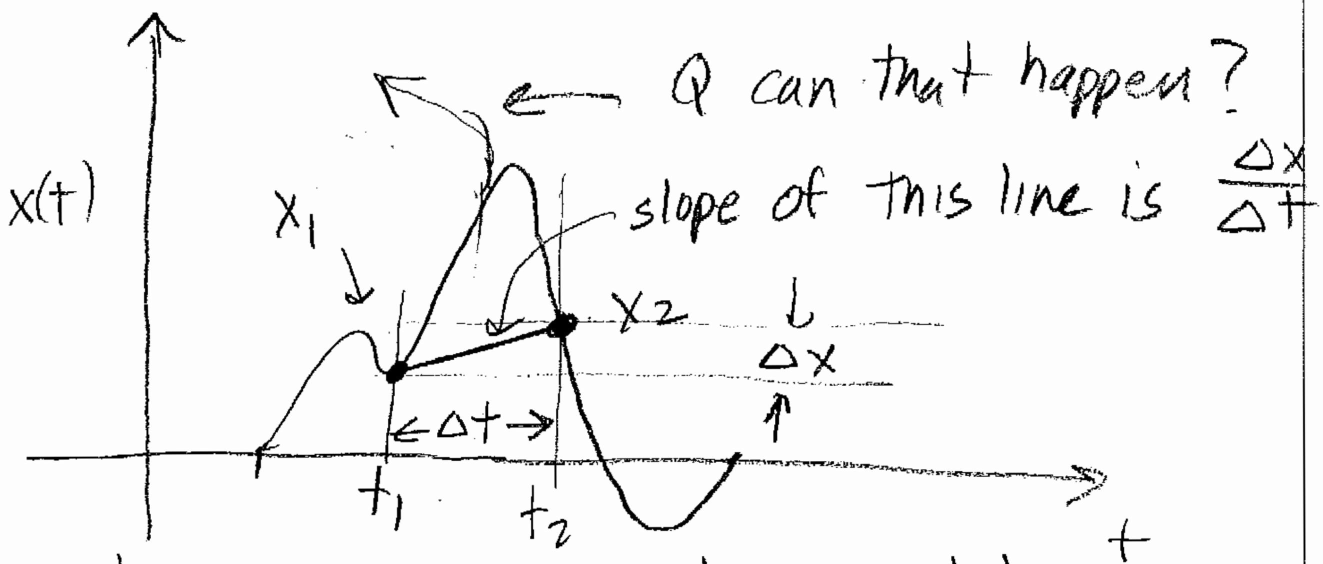
$$\vec{r}_1{}' = x_1'\hat{\imath} + y_1'\hat{\jmath} + z_1'\hat{k}$$

$$\vec{r}_1{}' + \vec{O} = \vec{r}_1$$

Could even have made primed system rotated w/r to original one.

Velocity and Acceleration

1-d .... x(t)



2 d! No vectors here!

$$\text{average velocity} \ldots \quad \bar{v} = \frac{\overbrace{x_2(t_2) - x_1(t_1)}^{\text{spatial displacement}}}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

dimensions: $[\bar{v}] = \frac{m}{s} \rightarrow$ slope on above graph

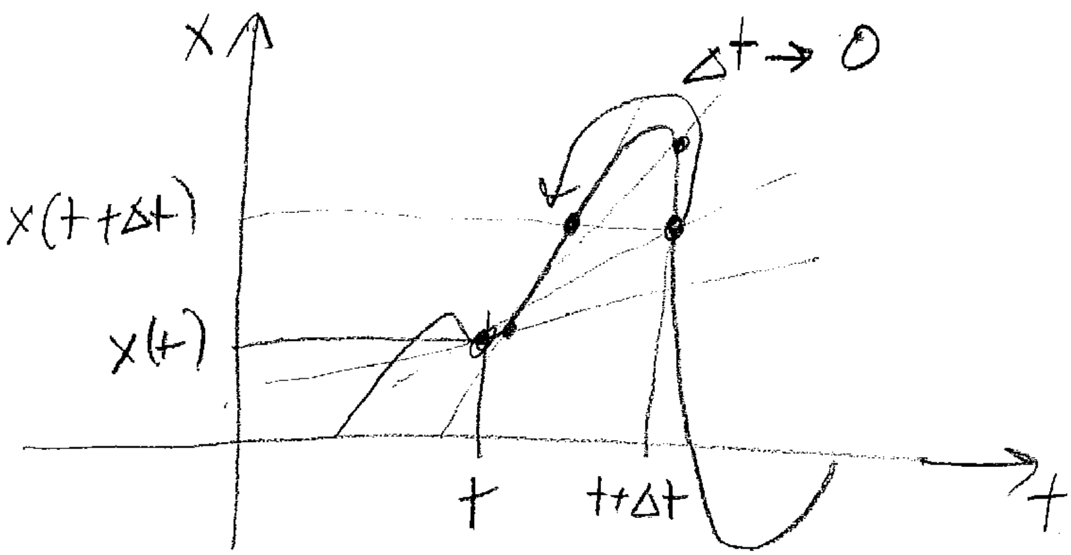
notation for dimensions

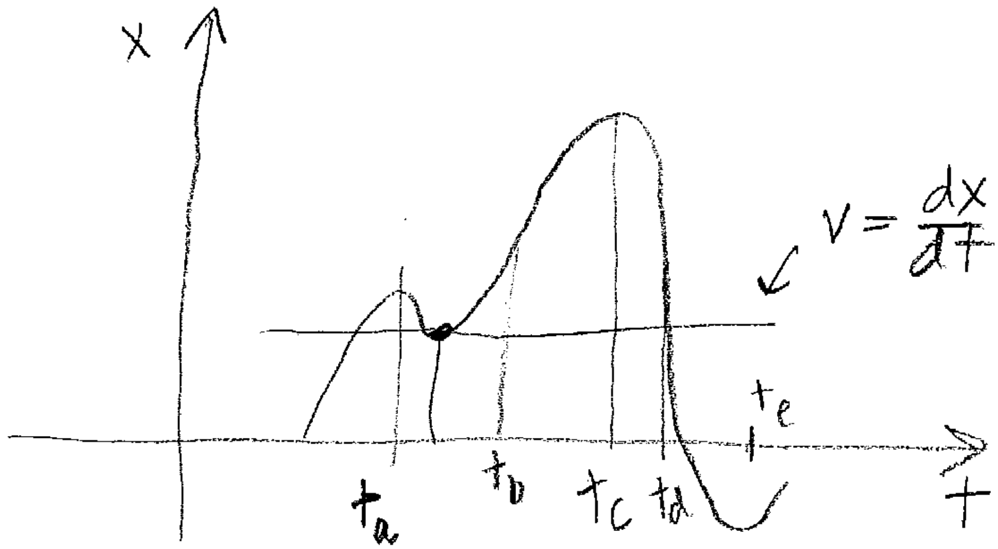
$\bar{v}$: can be ... negative, 0, positive traditionally ... change variables

$t_1 \to t \qquad x_1 \to x(t)$

$t_2 \to t+\Delta t \qquad x_2 \to x(t+\Delta t)$

let $t_2 \to t_1$ means let $\Delta t \to 0$



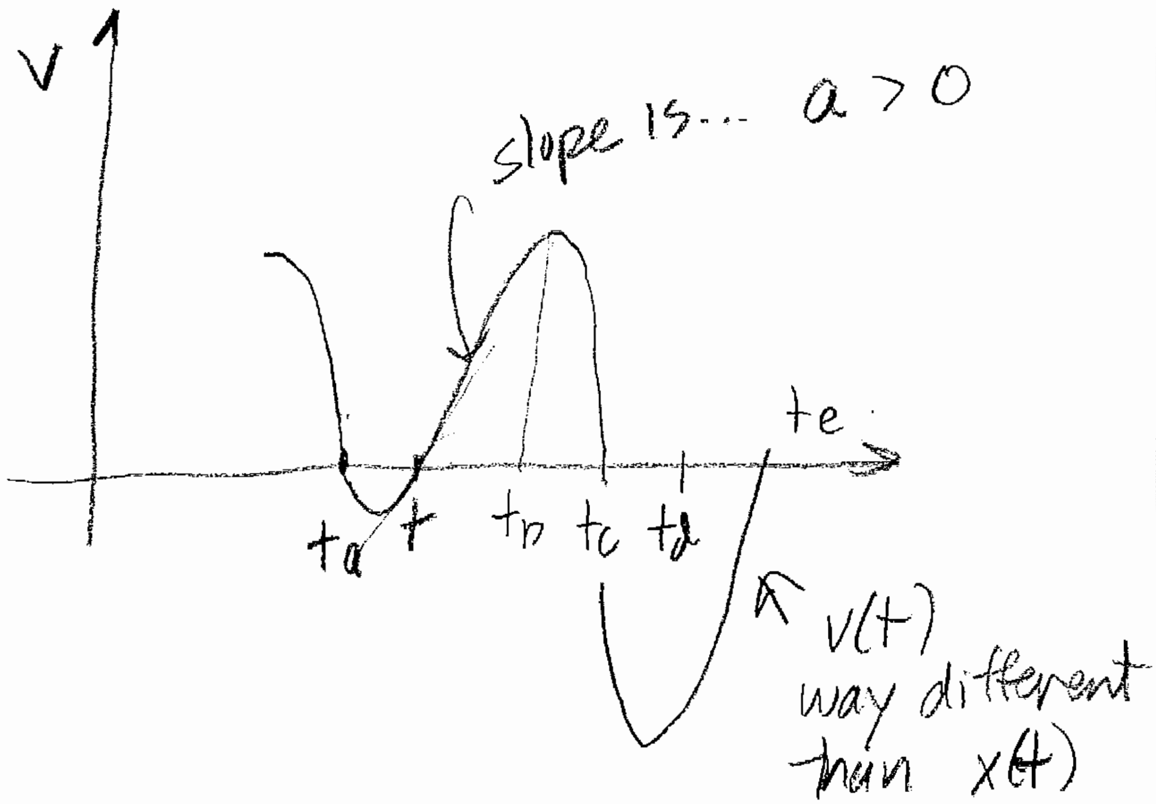


velocity $v = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$; speed $= |v|$

above $\simeq 0$! $\to$ $x(t) \neq 0$
$v(t) \sim 0$

acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ ... is that 0 above?

look at $v(t)$ ...



Into Two Dimensions (≈ p. 14–19 K+K)

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

or $\underbrace{[x(t), y(t)]}_{}$

"parametric" description of position → "the parameter" is t

$$\vec{v}(t) = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j}$$ ... not as easy to