Vector in 2-d: $\underset{\text{easy } (\geq 0)}{\underline{\text{magnitude}}}$ + $\underline{\text{direction}}$ that is harder

can be defined in a variety of ways

→ North, South, East, West

NE, NW, SE, SW

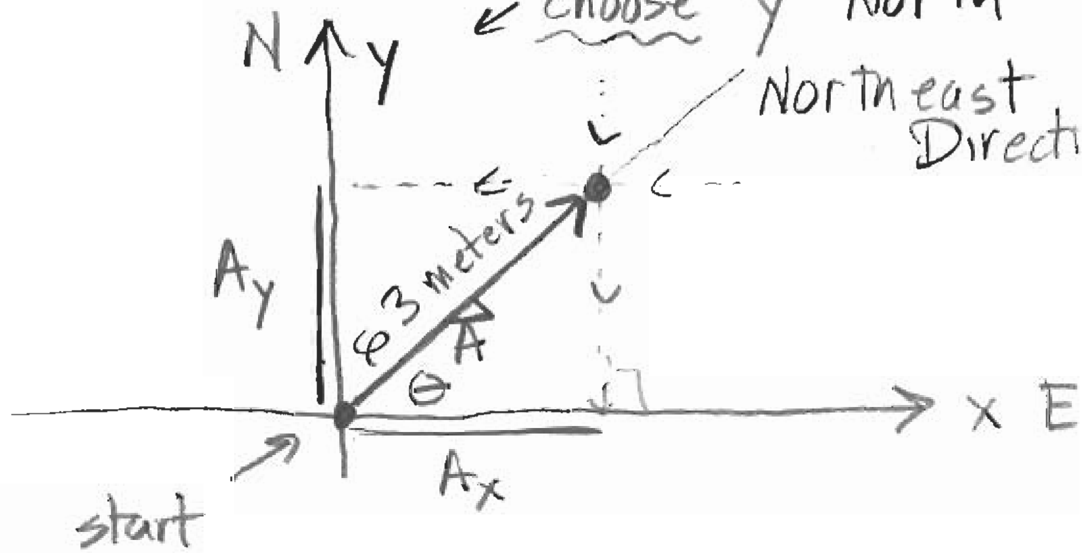
NNE, ENE, etc ...

→ angle w/r to an axis

→ $\boxed{\text{components}}$ ← in 1-d, was one component!

A vector $\vec{A}$ describes a $\underline{\text{displacement}}$ (not position) from the origin to a point 3 meters Northeast. Show the vector on an x-y plot.

← choose y North



Direction .. also, $\theta = 45^0$ w/r to the x-axis, counterclockwise = positive

$\varphi = -45^0$ w/r to y-axis

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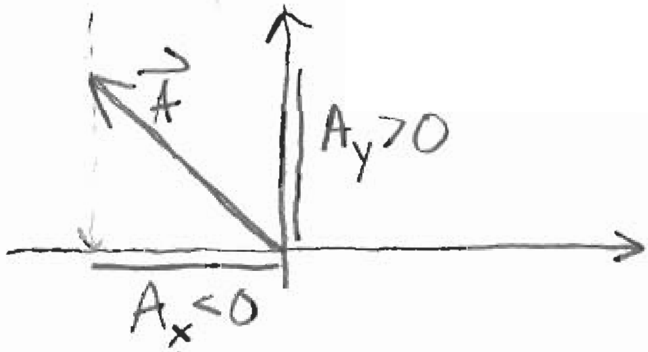
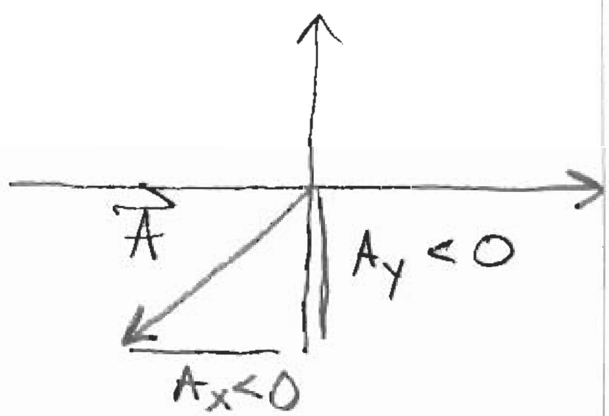
Components "shadow" of $\vec{A}$ along x and y axis .. called A_x and A_y

Possible for $A_x < 0$ and/or $A_y < 0$ for a different vector ...



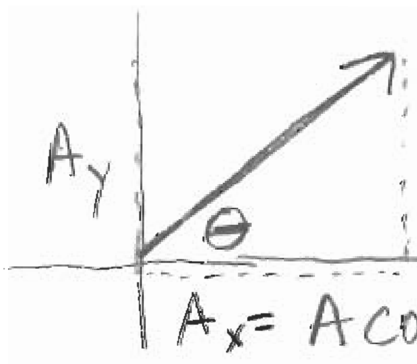


4 quantities: $\vec{A}$, $\underbrace{|\vec{A}| = A}_{\text{vector magnitude} \geq 0}$, $\underbrace{A_x, A_y}_{\text{vector components can be negative real numbers}}$

vector

Traditionally, θ is angle with respect to the x-axis (counterclockwise)

then



$A_y = A\sin\theta$

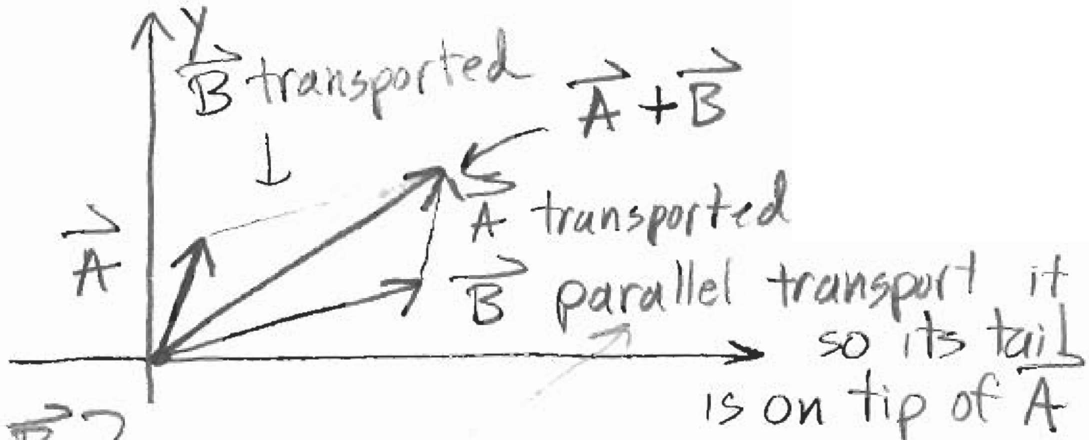
$A_x = A\cos\theta$

θ	A_x	A_y
$0 \to 90°$	≥ 0	≥ 0
$90 \to 180°$	≤ 0	≥ 0
$180 \to 270°$	≤ 0	≤ 0
$270 \to 360°$	≥ 0	≤ 0

Addition of 2-d Vectors

→ Geometric ... tip to tail

→ Algebraic ... add components



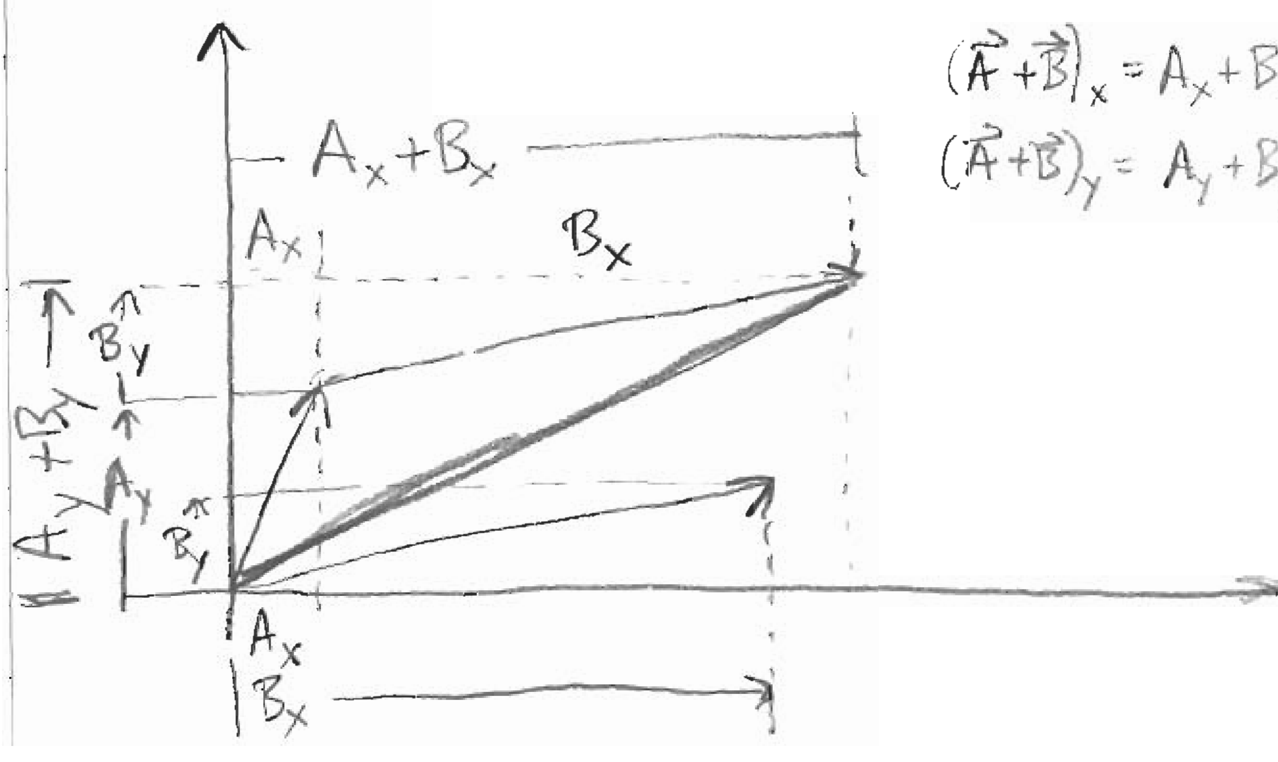
$\vec{A}+\vec{B}$?

could have done $\vec{B}+\vec{A}$ ... same result.

Nice Java Applet on Physics 21 Web Syllabus.

Examine components of all 3 ...

$$(\vec{A}+\vec{B})_x = A_x + B_x$$
$$(\vec{A}+\vec{B})_y = A_y + B_y$$



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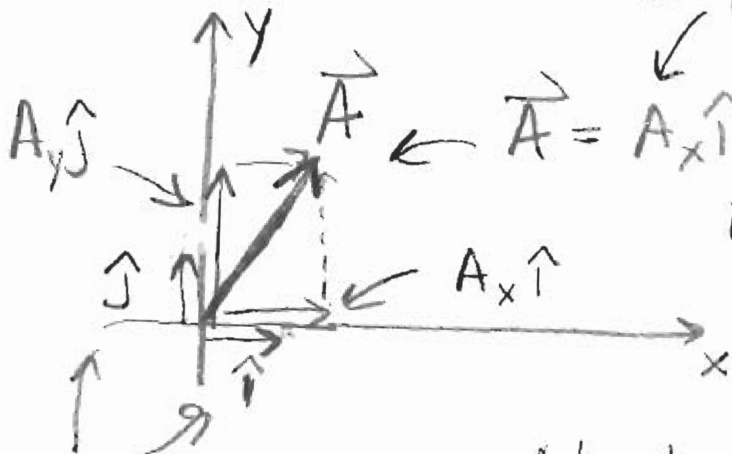
Put vector addition + components + unit vectors...

components



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

unit vectors

unit vectors in respective directions, x+y

Algebraic Vector Addition.

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

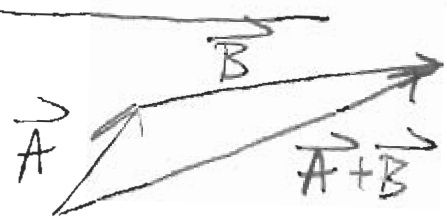
$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

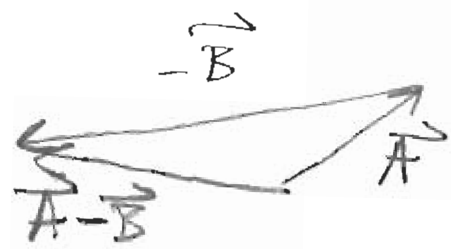
$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Things that work in 2-d like 1-d.

- multiplication by -1 flips direction
- multiplication by scalar...
  - $\rightarrow\ > 0$, changes length, not direction
  - $\rightarrow\ < 0$ ... " , + reverses direction

subtraction. $\vec{A} - \vec{B} = \vec{A} + (-1)\cdot\vec{B}$





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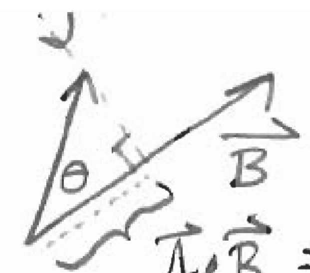
Scalar or "Dot" Product of Vectors

→ dull in 1-d, just the same as multiplication by real numbers

→ gets fun in 2-d, 3-d ... etc

Concept ... the <u>projection</u> of one vector on another .. times the "another's" length

$\vec{A}$



$$\vec{A}\cdot\vec{B} = (|\vec{A}|\cos\theta)\cdot|\vec{B}| = AB\cos\theta$$

(trig)

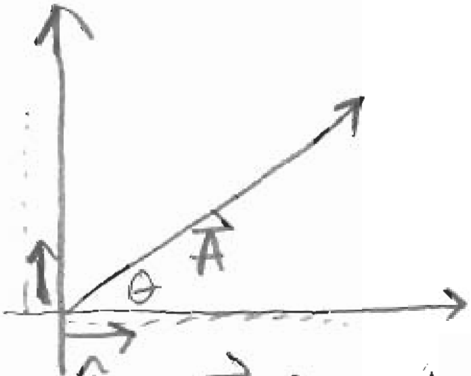
$\vec{B}\cdot\vec{A}$? $\vec{B}\cdot\vec{A}$

$$\vec{B}\cdot\vec{A} = (|\vec{B}|\cos\theta)\cdot|\vec{A}| = AB\cos\theta$$

so ... $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Relationship to components

$\vec{A}\cdot\hat{\jmath}$



$$\vec{A}\cdot\hat{\imath} = (A\cos\theta)\cancel{|\hat{\imath}|}^{1} = A_x$$

$$\vec{A}\cdot\hat{\jmath} = A\sin\theta\,\cancel{|\hat{\jmath}|}^{1} = A_y$$

note ..

$\hat{\imath}\cdot\hat{\imath} = 1$

$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{\imath} = 0$

$\hat{\jmath}\cdot\hat{\jmath} = 1$