Physics 21  H. Nelson  UCSB  

**Vectors**

**Topics**

- **Definition**
  - Manners of Description
    - magnitude and direction
    - components, unit + base vectors
  - "position" of a vector is elusive
    - translation usually conceptually OK
    - occasionally vector is "nailed down"
- **Addition**
  - geometric → "head to tail"
  - components
- **Multiplication by a scalar**
  - scalar can be negative, which reverses direction of the vector
- **Products of 2 Vectors**
  - Dot Product or "Scalar" Product
    - Geometric
    - Algebraic
    - "How much of one vector is along direction of another?"
  - Cross Product or "Vector" Product, "3d"
    - Geometric
    - Algebraic
    - "How much of one vector is 1 to direction of another?"

- Tensor Productbrief mention
Vector...a quantity with both magnitude and direction
"a directed line segment"
Not just a real number, because of direction

notation: books - \( \mathbf{A} \) (bold-face \( \mathbf{A} \))
writing - \( \vec{A} \) \( \Rightarrow \) arrow on top indicates direction
magnitude: \( |\vec{A}| = \text{positive (or 0) real number} \)

Vectors can live in any number of dimensions.
Spatial dimensions are special because we have intuitive ideas about rotations and translations; spatial vectors transform under rotations.

One Dimension (horizontal)

\( \vec{A} \) \( \Rightarrow \) magnitude is
\( |\vec{A}| \)
direction \( \Rightarrow \) left to right \( \approx \) adequate! don't need to be fancy!
only one other direction: right to left.
Multiplying by $-1$ reverses direction of a vector

\[ \vec{A} \]

\[ -(1) \cdot \vec{A} = -\vec{A} \]

“Parallel Translation”

(maintain direction)

+ length, but move

\[ -\vec{A} \]

\[ \text{a vector } \vec{A} \]

(b) Multiplying by a real number (a.k.a. a scalar) does the following:

1. changes the magnitude to $|b| |\vec{A}|$
2. if $b < 0$, reverses direction of $\vec{A}$

(still in one dimension!)

\[ \frac{1}{2} \vec{A} \]

\[ -3 \vec{A} \]

Adding Vectors: $\vec{A}$ and $\vec{B}$, to get $\vec{A} + \vec{B}$, do parallel translation of $\vec{B}$ so its tail is on top of $\vec{A}$’s tip

“Elephant Walk”
In one dimension, this is kind of trivial.

\( \overrightarrow{A} \rightarrow \text{like real number } A \)

\( \overrightarrow{B} \rightarrow \text{like real number } B \)

\( \overrightarrow{A} + \overrightarrow{B} \rightarrow \text{like } A + B \)

**note:** \( \overrightarrow{A} + (-\overrightarrow{A}) = \overrightarrow{0} \) or \( 0 \) as expected.

Unit vector \( \hat{A} \) can be constructed from any non-zero vector.

To do it: \( \hat{A} = \frac{\overrightarrow{A}}{|\overrightarrow{A}|} = \text{unit vector in } \overrightarrow{A} \text{ direction} \)

\( \hat{A} \) \( \overrightarrow{A} \) has no dimensions!

\( |\text{unit vector}| = 1 = \frac{|\overrightarrow{A}|}{|\overrightarrow{A}|} = 1 \)

In one dimension, only has 2 possible directions!
Left to Right, Right to Left

\[ \hat{A} \rightarrow \hat{A} \]
\[ \hat{B} \rightarrow \hat{B} \]

Base Vectors. (1 dimension still)

→ hold up a number line to your vector... like a ruler

\[ \hat{A} \]

\[ \begin{array}{cccccccc}
-1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \]

axis

#'/s' = kind of arbitrary

. when \( \hat{A} \) is a displacement, meters etc
. \( \hat{A} \) is a velocity, m/s etc
. an acceleration, m/s² etc

useful to define a unit vector in the direction of increasing \( x \) ... call that

\[ \hat{i} \]

when handwritten, \( \hat{i} \) is boldface

in text

then...

\[ \hat{A} = A_x \hat{i} \rightarrow \text{defines direction} \]

magnitude + or - \( A_x \) called "x-component"
Two Dimensions

Now imagine, say, a plane. Need two axes to describe points in a plane... and 2-d vectors.

simplest system has axes perpendicular

origin? For vectors, not always pertinent, because "parallel translation".

Rotations very important in 2-d.

This one is different... $x$" to $y$" is clockwise, not counterclockwise.
We use "right handed" coordinate systems (arbitrary but universal convention)

→ x rotated to y goes \textbf{counterclockwise}

→ take right hand, make it flat, thumb up:

\begin{itemize}
  \item [\textbf{Thumb}]
  \item [index]
  \item [king]
  \item [pinky]
\end{itemize}

→ align four fingers in x-direction, so that when you bend them, they bend toward y-axis

\[ X \bullet \quad \text{Right Hand} \]

Thumb sticks up, out of page... that is a right-handed coordinate system... had you used left hand, then your thumb would have poked down into page.

(If you look at a left handed coordinate system from under the page, it looks right handed! Hmm... handedness is related to 3 dimensions!)