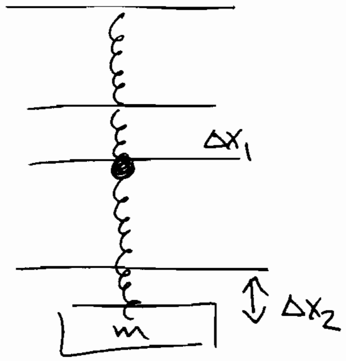
Ph 21 HW #6 Solutions

2.31

(a)



Imagine the stretched spring immediately before mass m is released.

The point of contact between the two springs is at a constant height $\Rightarrow a_{contact} = 0$

<u>Newton's 2nd Law</u>

$$\sum F_y = k_1 \Delta x_1 - k_2 \Delta x_2 = m a_y = 0$$

$$\therefore \boxed{k_1 \Delta x_1 = k_2 \Delta x_2}$$

If Δx_1 is length first spring is stretched,

Δx_2 " " second spring "

$\Delta x = \Delta x_1 + \Delta x_2$ is length ~~of~~ the "effective" spring is stretched.

$$\sum F = \underbrace{-k_2 \Delta x_2}_{\text{Real forces}} = \underbrace{-K_{eff} \Delta x}_{\text{Treat as a single spring with "effective" spring constant } K_{eff}}$$

$$\Rightarrow K_{eff} = k_2 \frac{\Delta x_2}{\Delta x} = \frac{k_2 \Delta x_2}{\Delta x_1 + \Delta x_2} = \frac{k_2}{(1 + \Delta x_1 / \Delta x_2)}$$

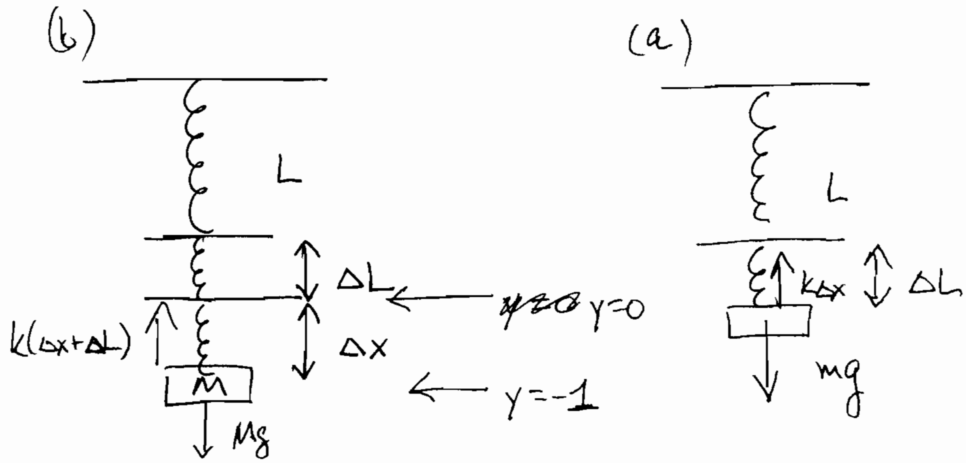
$$\boxed{K_{eff} = \frac{k_1 k_2}{k_1 + k_2}}$$

$$\omega_a = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

For $k_1 = k_2 = m$

$$\boxed{\omega_a = \sqrt{\frac{k}{2m}}}$$

(1)



(a) <u>Newton's 2nd Law</u>

$$\sum F = m a_y = 0 \quad \text{(at rest)}$$

$$\sum F = k\Delta L - mg = 0$$

$$\boxed{\Delta L = \frac{mg}{k}} = (1\,\text{gm})\left(\frac{10\ \text{m/s}^2}{1\ \text{N/m}}\right) = .01\,\text{m}$$

$$\boxed{\Delta L = 1\ \text{cm}}$$

(b) <u>Newton's 2nd Law</u>

$$\sum F = k(\Delta x + \underbrace{\Delta L) - mg}_{=0 \text{ using part (a)}}$$

$$= k\Delta x$$

(c) Maximum speed is at $y=0$ since all potential energy is converted into kinetic energy.

Time $\Rightarrow \omega t = \pi/2$, $\boxed{t = (\pi/2\omega) = 0.05\,\text{s}}$

The motion is still given by

$y(t) = A\cos(\omega t)$ with $\omega = \sqrt{k/m}$

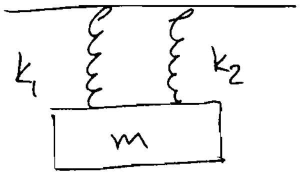
$A = -1$ for this part

$$v(t) = \dot{y}(t) = -A\omega\sin(\omega t)$$

at $\omega = \pi/2$

$$v(t) = (+1\,\text{cm})\sqrt{\frac{k}{m}} = +1\,\text{cm}\sqrt{\frac{1\ \text{N/m}}{.0001\ \text{kg}}} = \boxed{31.6\ \frac{\text{cm}}{\text{s}}}$$

2.31
(b)



Newton's 2nd Law

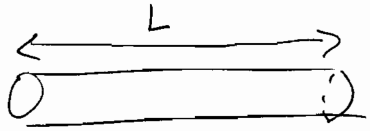
$$\sum F = \underbrace{(k_1 + k_2)}_{= K_{eff}} \Delta x = m a_y$$

$$\omega_0 = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{(k_1+k_2)}{m}}$$

For $k_1 = k_2 = m$

$$\boxed{\omega_0 = \sqrt{\frac{2k}{m}}}$$

(3.1)



Density $\rho = \frac{\rho_0 x^2}{\ell^2}$

<u>Total Mass</u>

$$M = \int_0^\ell \rho \, dx$$

$$= \int_0^\ell \rho_0 \frac{x^2}{\ell^2} \, dx$$

$$= \frac{\rho_0 \ell}{3}$$

<u>Center of Mass</u>

$$\overline{X} = \frac{1}{M} \int_0^\ell x \rho \, dx$$

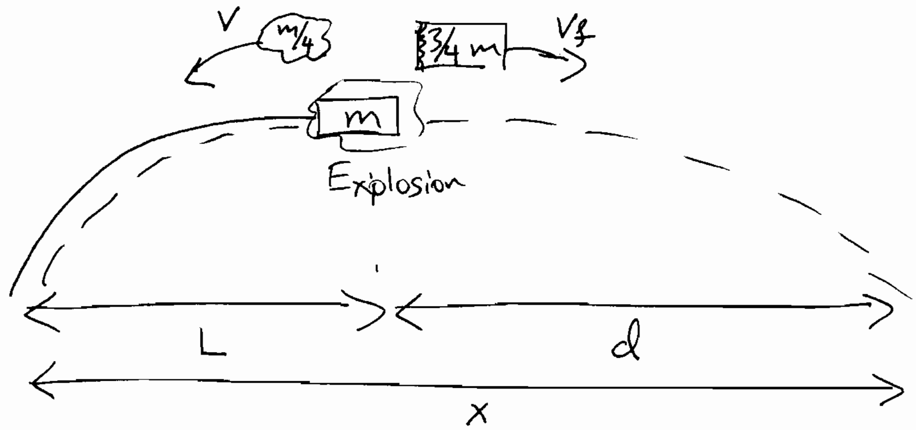
$$= \frac{1}{M} \int_0^\ell \rho_0 \frac{x^3}{\ell^2} \, dx$$

$$= \frac{1}{M} \frac{\rho_0 \ell^2}{4}$$

$$= \frac{\rho_0 \ell^2}{\rho_0 \ell} \left(\frac{3}{4} \right)$$

$$\boxed{\overline{X} = \frac{3}{4} \ell}$$

3.4



At the top of the projectile's trajectory, $v_y = 0$
During the explosion kinetic energy is not conserved
(for example chemical energy gets turned into kinetic energy) but
linear momentum is conserved.

$$\Rightarrow \quad mv = -\frac{m}{4}v + \frac{3}{4}mv_f$$

$$\therefore \quad \boxed{v_f = \frac{5}{3}v}$$

(for the smaller piece to return to the launch point it must have velocity $-v$)

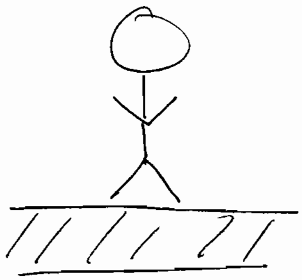
The additional distance, d, travelled by the larger piece is given by

$$\frac{d}{L} = \frac{v_f T}{v T} = \frac{v_f}{v}$$

$$d = 5/3\, L$$

$$\Rightarrow \quad \boxed{x = \frac{8}{3}L}$$

3.8



$$\text{Impulse} = \Delta p = p_f - p_i$$

where p_f is the linear momenta of the woman $\underline{\text{immediately}}$ after leaving the ~~groung~~ ground.

$p_i = 0$ is her initial momenta

To find p_f, note that her initial KE equals her PE at top of jump

$$KE_{initial} = \frac{1}{2}mv^2 = \boxed{\frac{p_f^2}{2m} = mgh} = PE_{top}$$

$$p_f = m\sqrt{2gh}$$

$$\boxed{I = \Delta p = m\sqrt{2gh}}$$

$$I = 50\,kg\sqrt{(2)\,9.8\,\frac{m}{s^2}\,(0.8\,m)}$$

$$= 198\ \frac{kg\cdot m}{s}$$