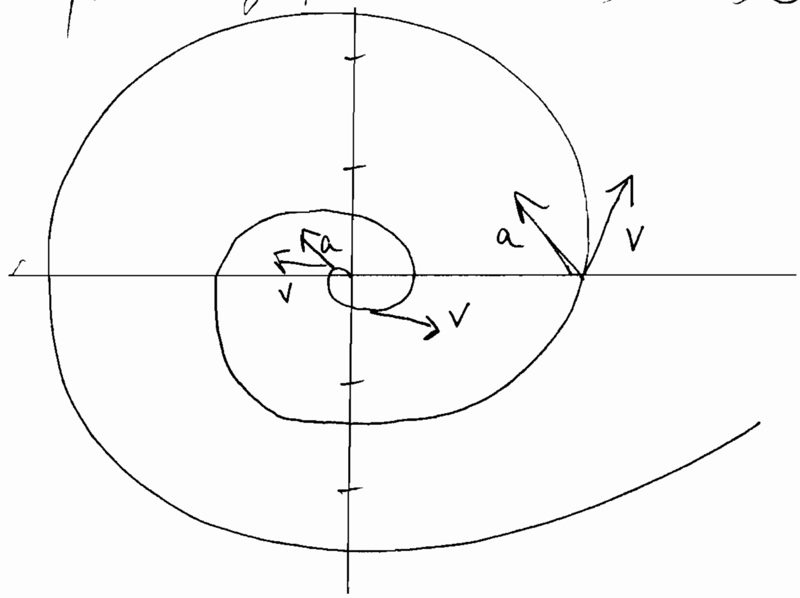
Physics 21 HW#5 Solutions

(1)
1.20

(a)



Initially the acceleration is pointed <u>outward</u> in the radial direction

Past $\theta = 1/\sqrt{2}$ the acceleration is pointed radially <u>inward</u>

$r = A\theta$ $\qquad$ $A = (1/\pi)$ m/rad

$\theta = \alpha t^2/2$

(b) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$

$\cancel{= \alpha A(1 - \alpha t^2)\hat{r} +}$

$(\alpha A - A\theta\,\alpha^2 t^2)\hat{r} + (A\theta\alpha + 2A(\alpha t)^2)\hat{\theta}$

$= A\alpha(1 - 2\theta^2)\hat{r} + A\alpha^2\left(\frac{t^2}{2} + 2t^2\right)\hat{\theta}$

$$\boxed{\vec{a} = A\alpha(1 - 2\theta^2)\hat{r} + \left(\frac{5}{2}A\alpha^2 t^2\right)\hat{\theta}}$$

So when $\theta = 1/\sqrt{2}$ rad $\quad a_r = (1 - 2(1/\sqrt{2})^2)A\alpha = 0$

(c) $\vec{a} = A\alpha\left[(1 - 2\theta^2)\hat{r} + (5\theta)\hat{\theta}\right]$

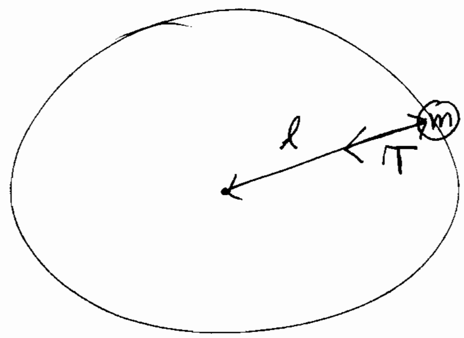
$a_r = a_\theta \Rightarrow \quad 1 - 2\theta^2 = 5\theta$

$2\theta^2 + 5\theta - 1 = 0$

$$\boxed{\theta = \frac{-5 \pm \sqrt{33}}{4}}$$

(2)

$l = 1m$



Mass feels a centripetal acceleration equal to g

The only force acting on the mass is the tension of the rope

<u>Newton's 2nd Law</u>

$$\sum F = T = ma = mg$$

For uniform circular motion

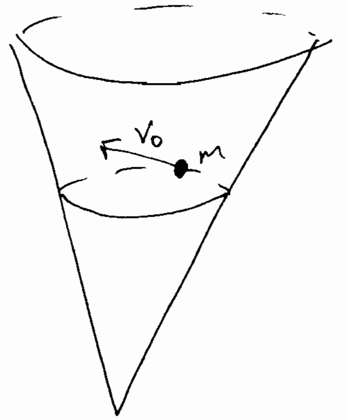
$$a = \frac{v^2}{l} \Rightarrow g = \frac{v^2}{l}$$

The time to complete one full orbit is

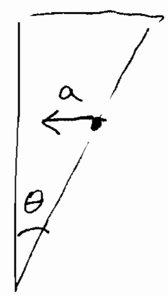
$$T = \frac{(2\pi l)}{v} = \frac{(2\pi l)}{\sqrt{gl}}$$

$$\boxed{T = (2\pi)\sqrt{\frac{l}{g}}} = 2.01s$$

2.9
(3)



Force Diagram (cross-sectional view)



Newton's 2nd Law

$$\sum F_x = -N\cos\theta = m a_x$$

$$\sum F_y = N\sin\theta - mg = m a_y = 0$$ (since the particle is at constant height)

$$\Rightarrow \quad N = \frac{mg}{\sin\theta}$$

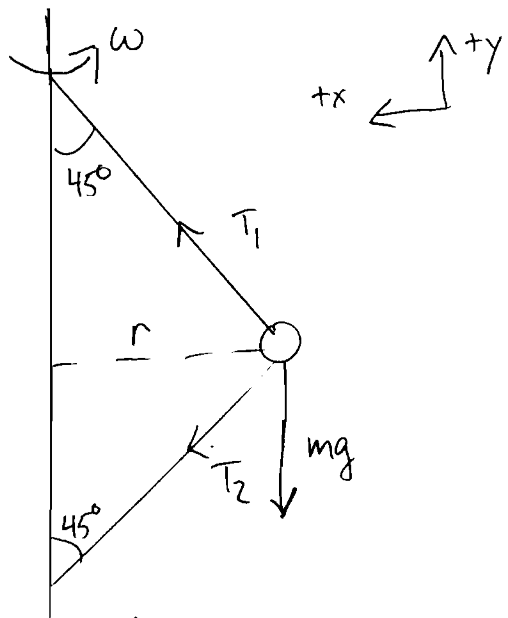
$$\therefore \quad \boxed{\cancel{m}a = -\cancel{m}g\cot\theta}$$

Uniform circular motion $\Rightarrow$ $a = -v_0^2/R$

$$\boxed{\frac{v^2}{R} = \cancel{m}g\cot\theta}$$

$$\boxed{R = \left(\frac{v^2}{\cancel{m}g}\right)\tan\theta}$$

2011
(4)



the radius of the circular orbit is

$$\boxed{r = \frac{\ell}{\sqrt{2}}}$$

Newton's 2nd Law

$$\begin{cases} (*) \quad \Sigma F_x = \dfrac{T_1}{\sqrt{2}} + \dfrac{T_2}{\sqrt{2}} = ma_x \\ (**) \quad \Sigma F_y = \dfrac{T_1}{\sqrt{2}} - \dfrac{T_2}{\sqrt{2}} - mg = ma_y = 0 \end{cases}$$

$a_y = 0$ since the mass is at a constant height

$$a_x = \frac{v^2}{r} = \omega^2 r = \frac{\omega^2 \ell}{\sqrt{2}}$$

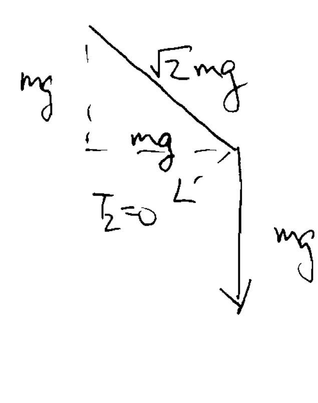
Adding (*) and (**)

$$\sqrt{2}\, T_1 = m\left(g + \frac{\omega^2 \ell}{\sqrt{2}}\right)$$

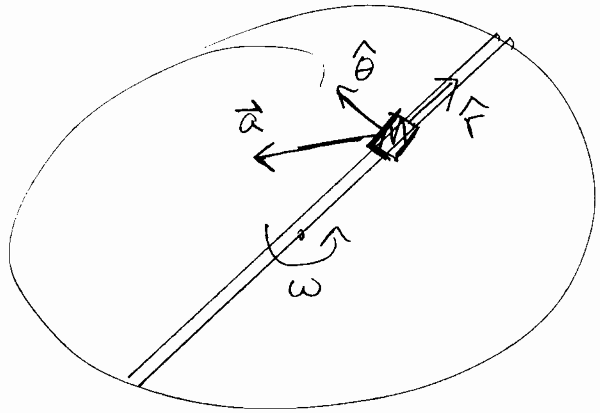
$$\boxed{T_1 = \frac{m}{\sqrt{2}}\left(g + \frac{\omega^2 \ell}{\sqrt{2}}\right)}$$

For $\omega^2 \ell = \sqrt{2} g$

$$\boxed{\begin{aligned} T_1 &= \sqrt{2}\, mg \\ T_2 &= 0 \end{aligned}}$$



(5)
2.29



Bird's eye view
$Mg = W$

(a)

$$\vec{a} = -r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$$
$$= -r\omega^2\hat{r} + 2\omega v_0\hat{\theta}$$

where $r(t) = v_0 t$

Newton's 2nd Law

$$\sum F = \mu N\hat{e}_f = m\vec{a}$$

(note: friction is in whatever direction needed to oppose acceleration call this direction $\hat{e}_f$

(b) The magnitude of the acceleration is

$$|\vec{a}| = \sqrt{\omega^4 v_0^2 t^2 + 4v_0^2\omega^2}$$
$$= \omega v_0\sqrt{4 + (\omega t)^2}$$

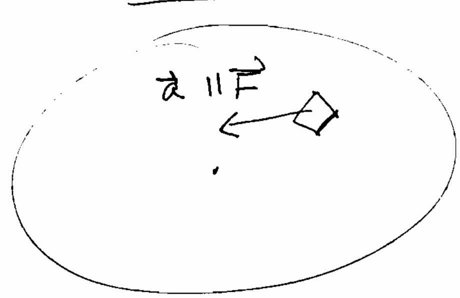
The car just starts to skid when the force of friction can no longer counteract the needed acceleration

$$\mu g \leq \omega v_0\sqrt{4 + (\omega t)^2}$$

$$\left(\frac{\mu g}{\omega v_0}\right)^2 \leq 4 + (\omega t)^2$$

$$\boxed{t = \frac{1}{\omega}\sqrt{\left(\frac{\mu g}{\omega v_0}\right)^2 - 4}}$$

(c)



$$\vec{F} = m\omega v_0\left(-\sqrt{\left(\frac{\mu g}{\omega v_0}\right)^2 - 4}\;\hat{r} + 2\hat{\theta}\right)$$

immediately before skidding