Physics 21 Problem Set 1 – Solutions

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Problem 1

(a,b)

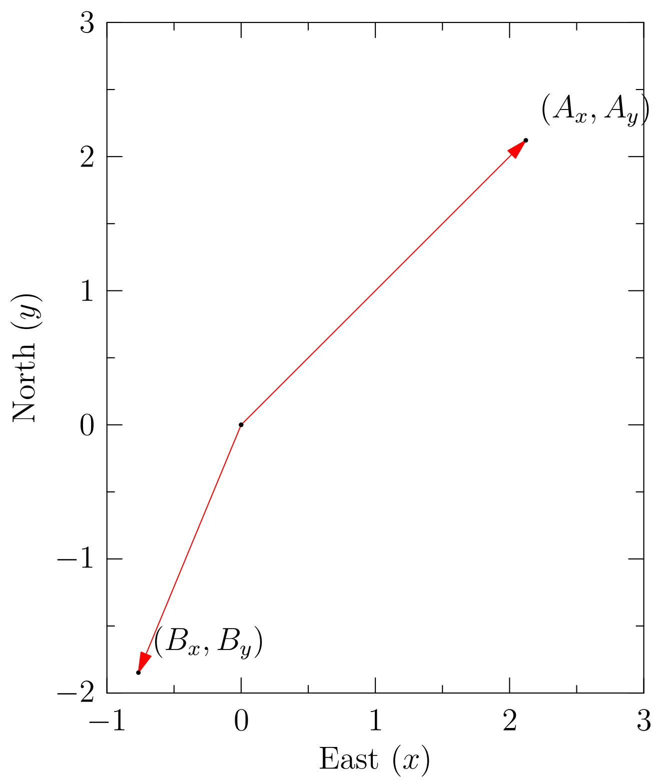

Figure 1: Vectors $\vec{A}$ and $\vec{B}$

(c)

$$A_x = 3\cos 45^\circ \approx 2.12$$
$$A_y = 3\sin 45^\circ \approx 2.12$$

(d)

$$\hat{A}_x = \cos 45° \approx 0.71$$
$$\hat{A}_y = \sin 45° \approx 0.71$$

(e)

South-Southwest is halfway between Southwest (225°) and South (270°), so it is at $(225 + 270)/2 = 247.5$ degrees.

$$B_x = 2\cos 247.5° \approx -0.77$$
$$B_y = 2\sin 247.5° \approx -1.84$$

$$\hat{B}_x = \cos 247.5° \approx -0.38$$
$$\hat{B}_y = \sin 247.5° \approx -0.92$$

(f,g)

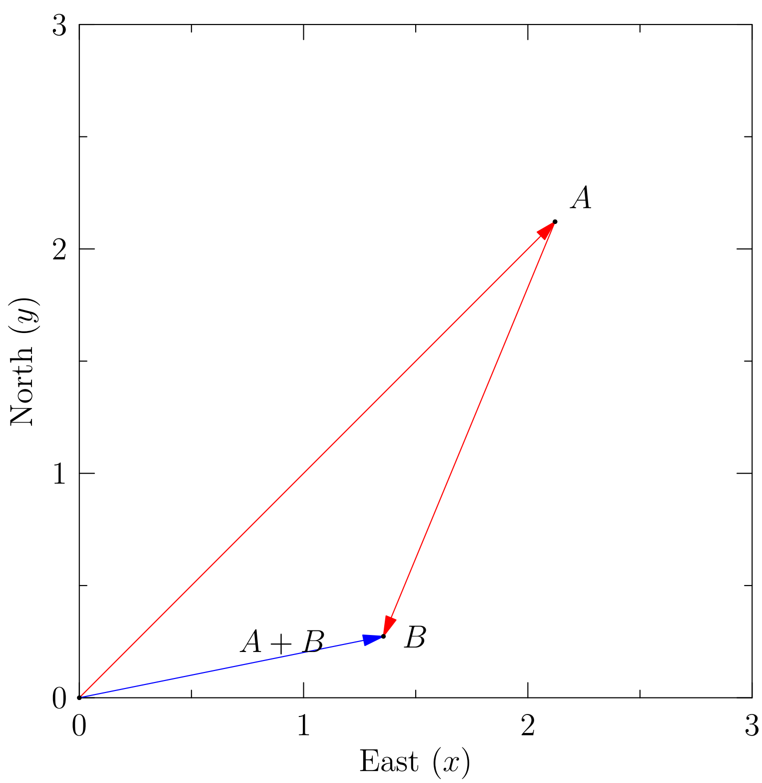

Figure 2: Vectors $\vec{A}$ and $\vec{B}$ summed using tip-to-tail method

$$
\begin{aligned}
(A+B)_x &\approx 2.12 - 0.77 = 1.35 \\
(A+B)_y &\approx 2.12 - 1.84 = 0.28
\end{aligned}
$$

(h)

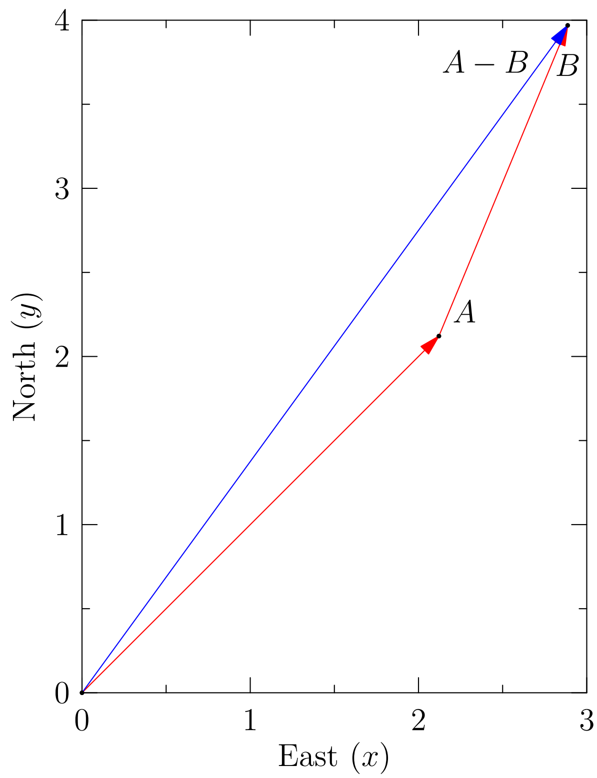

Figure 3: $\vec{A}$ minus $\vec{B}$ using tip-to-tail method

$$
\begin{aligned}
(A-B)_x &\approx 2.12 - (-0.77) = 2.89 \\
(A-B)_y &\approx 2.12 - (-1.84) = 3.96
\end{aligned}
$$

(i)

To find the smaller angle between $\vec{A}$ and $\vec{B}$ use $247.5° = -112.5°$, so the smaller angle is

$$\theta = 45° - (-112.5°) = \boxed{157.5°}$$

(j)

$$\vec{A} \cdot \vec{B} = AB\cos\theta = (2)(3)\cos 157.5° = \boxed{-5.54}$$

(k)

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(0.707)(-0.765)+(0.707)(-1.848)}{(2)(3)} = -0.92$$

Using $\cos^{-1}$, we get that

$$\boxed{\theta = 157.5^\circ}$$

using this method, agreeing wiht part (j).

(l)

The component of $\vec{A}$ in the direction of $\vec{B}$ is

$$\vec{A}|_B = \vec{A}\cdot\hat{B} = -2.77.$$

In coordinates, this is

$$(\vec{A}|_B)\hat{B} = -2.77(-0.38,-0.92) = \boxed{(1.06, 2.56)}$$

(m)

The component of $\vec{B}$ in the direction of $\vec{A}$ is

$$\vec{B}|_A = \vec{B}\cdot\hat{A} = -1.85.$$

In coordinates, this is

$$(\vec{B}|_A)\hat{A} = -1.85(0.707, 0.707) = \boxed{(-1.31, -1.31)}$$

Problem 2 (KK 1.2)

The cosine of the angle between

$$\vec{A} = (3\hat{i}+\hat{j}+\hat{k}) \qquad \vec{B} = (-2\hat{i}-3\hat{j}-\hat{k})$$

is

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

where

$$\vec{A}\cdot\vec{B} = (3)(-2)+(1)(-3)+(1)(-1) = -10$$

$$A = \sqrt{3^2+1^2+1^2} = \sqrt{11} \qquad B = \sqrt{2^2+3^2+1^2} = \sqrt{14}$$

so

$$\cos\theta = \frac{-7}{\sqrt{11}\sqrt{14}} = \boxed{-0.805}$$

Problem 3 (KK 1.4)

If

$$\|\vec{A} - \vec{B}\| = \|\vec{A} + \vec{B}\|,$$

then by squaring both sides,

$$\begin{aligned}
(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) &= (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) \\
\|A\|^2 - 2\vec{A} \cdot \vec{B} + \|B\|^2 &= \|A\|^2 + 2\vec{A} \cdot \vec{B} + \|B\|^2 \\
-2\vec{A} \cdot \vec{B} &= -2\vec{A} \cdot \vec{B} \\
\vec{A} \cdot \vec{B} &= 0
\end{aligned}$$

so $\vec{A}$ and $\vec{B}$ are orthogonal (perpendicular).