Stress (Tension / Compressive) and Strain

Section 14-5
RHK4

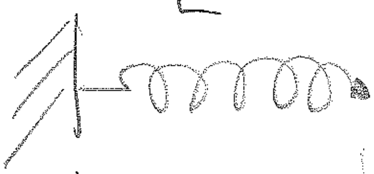
Stress → force / area
(like a pressure)

Strain → $\Delta L / L$

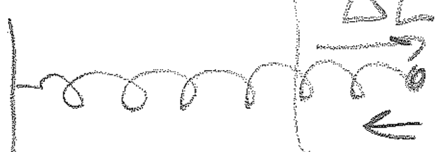
Idea: most every thing, for small $\Delta L/L$, is a spring

L

equilibrium.

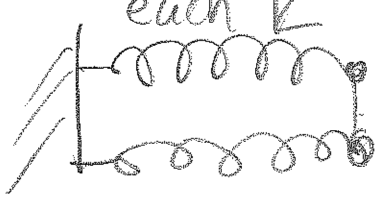


ΔL

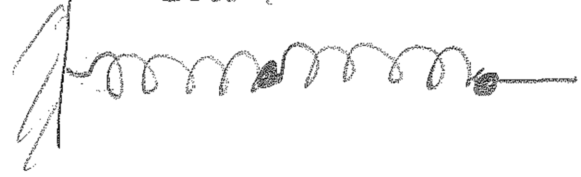


$F = -k\Delta L$

each k

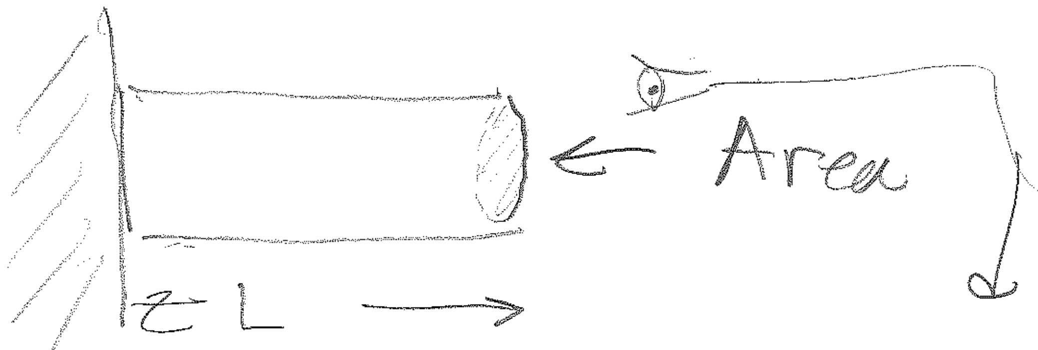


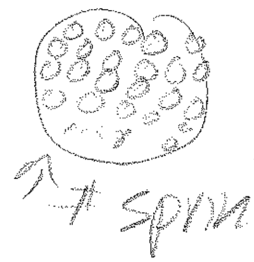
each



"$k_{parallel}$" A > "k_{series}"
B =
$k_{parallel} = 2k$ C < $k_{series} = \frac{1}{2}k$

Slab of "stuff"





$K \propto \text{Area}$

$K \propto \frac{1}{L}$

$$F = -E \frac{A}{L} \Delta L$$

Young's Modulus

N/m^2

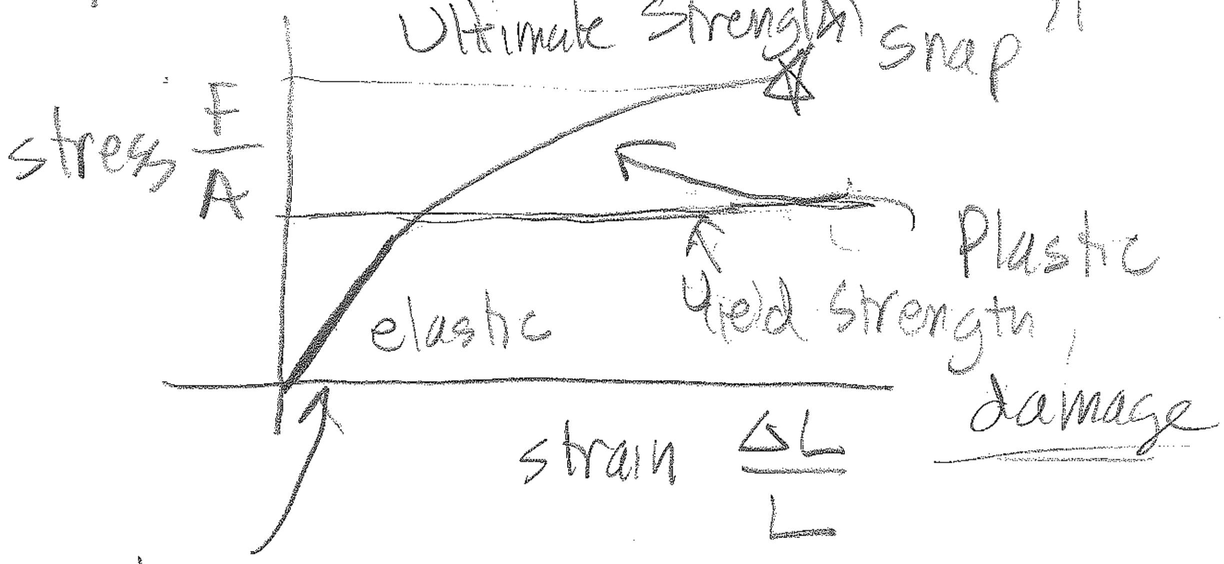
usually written

$$\frac{F}{A} = E \frac{\Delta L}{L}$$

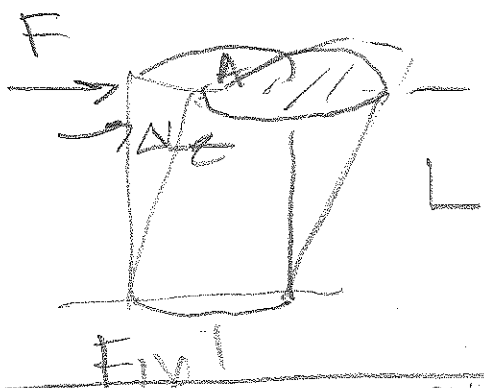
stress — "understood" — strain

$E \sim 9 \cdot 10^9 \ N/m^2$ (bone)

only an approximation ---



linear range (Springy!)

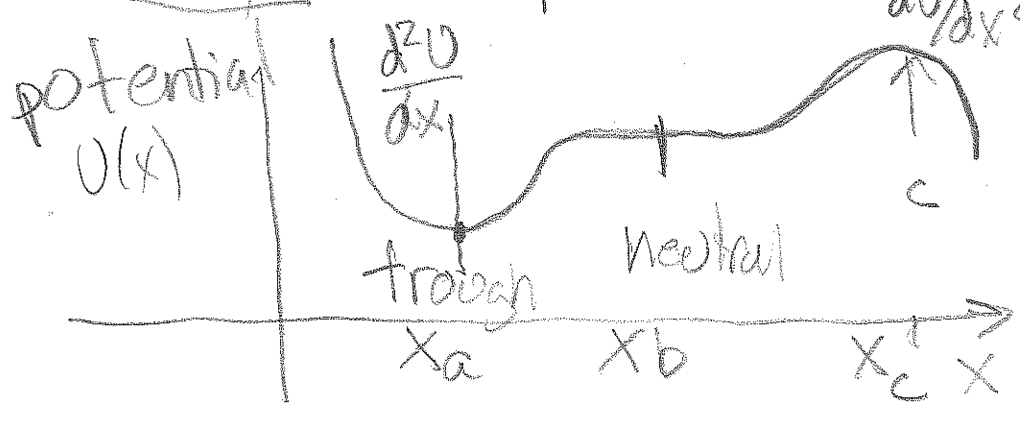


"Sheer"

$\frac{F}{A} = (\text{Sheer modulus}) \frac{\Delta L}{L}$

$\boxed{\text{PHYS } 14\text{-}4}$

Stability map $U(x)$ or $U(x,y)$



all points:

$\frac{dU}{dx} = 0$

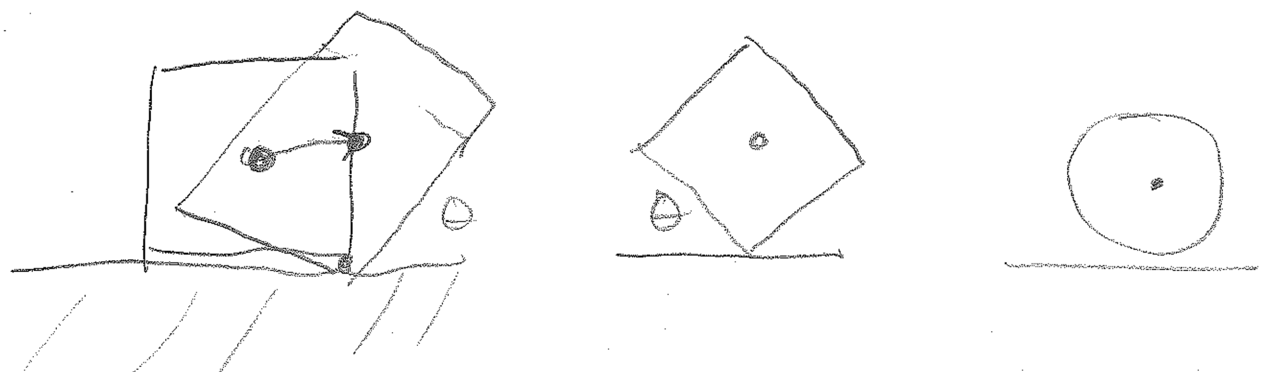
$F = -\frac{dU}{dx} = 0$

(A) $\left.\frac{d^2U}{dx^2}\right|_{x_a} < 0, \quad \left.\frac{d^2U}{dx^2}\right|_{x_b} = 0, \quad \left.\frac{d^2U}{dx^2}\right|_{x_c} > 0$

(B) $\left.\frac{d^2U}{dx^2}\right|_{x_a} > 0, \quad \left.\frac{d^2U}{dx^2}\right|_{x_b} = 0, \quad \left.\frac{d^2U}{dx^2}\right|_{x_c} < 0$

(C) Both wrong

Book / ball . . . as a function of θ



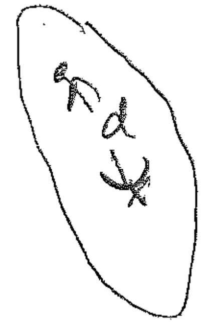
cm goes up.

$\frac{d^2U}{d\theta^2} > 0$ $\qquad$ $\frac{d^2U}{d\theta^2} < 0$ $\qquad$ $\frac{d^2U}{d\theta}$

what "force" depends on θ ?

$\frac{dU}{d\theta}$ will be TORQUE

Physical Pendolum



cm

$ML^2 \to I \qquad L \to d$

$$I\ddot{\theta} \cong -Mgd\theta$$

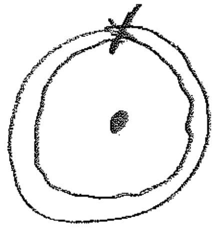
simple

$$\omega = \sqrt{\frac{Mgd}{I}} \qquad \Big| \qquad \sqrt{\frac{g}{L_0}}$$

$$L_0 = \frac{I}{Md}$$

center of oscillation

Ring : suspend on edge



$$I = MR^2 + MR^2$$
$$= 2MR^2$$

$$\omega = \sqrt{\frac{\cancel{M}gR}{2\cancel{M}R^2}} = \sqrt{\frac{g}{2R}}$$

$$L_0 = \frac{2MR^2}{M \cdot R} = 2R$$

$$ML^2\ddot{\theta} = -MgL\theta$$

$$m\ddot{x} = -kx \rightarrow$$ solution

$x_m \cos(\omega t+\phi)$

$x_m \sin(\omega t+\phi)$

$$\omega = \sqrt{\frac{k}{m}},\ T = 2\pi\sqrt{\frac{m}{k}}$$

$$\theta = \theta_m \cos(\omega t+\phi).$$

period $T =$

(A) Can't tell

(B) $2\pi\sqrt{\dfrac{M}{MgL}} = 2\pi\sqrt{\dfrac{1}{gL}}$

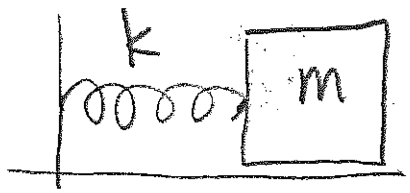
(C) $2\pi\sqrt{\dfrac{MgL}{ML^2}} = 2\pi\sqrt{\dfrac{g}{L}}$

(D) $2\pi\sqrt{g}$

(E) $2\pi\sqrt{gL}$

Simple Harmonic Motion... Damped + Driven

Note 10.1 KK
10.2 KK 15-8
10.3 KK 15-9



$$m a_x = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

Full solution needs: initial position (x_0) and velocity (v_0)

variables

$\boxed{\text{plug in!}}$ $$\underline{x(t) = x_m \cos(\omega t + \phi)} \qquad \omega = \sqrt{\frac{k}{m}}$$

$$x_0 = x(0) = x_m \cos(\omega \cdot 0 + \phi)$$

$$x_0 = x_m \cos\phi$$

$$v_0 = \left.\frac{dx}{dt}\right|_{t=0} = -\omega x_m \sin(\omega t + \phi)\Big|_{t=0}$$

$$v_0 = -\omega x_m \sin\phi$$

Use to evaluate ϕ, x_m

$$-\frac{v_0}{\omega} = x_m \sin\phi \qquad x_0 = x_m \cos\phi$$

$$\frac{x_m \sin\phi}{x_m \cos\phi} = \tan\phi = -\frac{v_0}{\omega x_0}$$

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solve: $\boxed{\phi = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right)}$

$$x_m^2 \sin^2\phi + x_m^2 \cos^2\phi = x_m^2 = \left(\frac{-v_0}{\omega}\right)^2 + x_0^2$$

$$\boxed{x_m = \pm\sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}}$$

can get ↗ from energy too!

Next: Add damping

Add driving

Need complex numbers

$\Rightarrow$ K+K Note 10.1

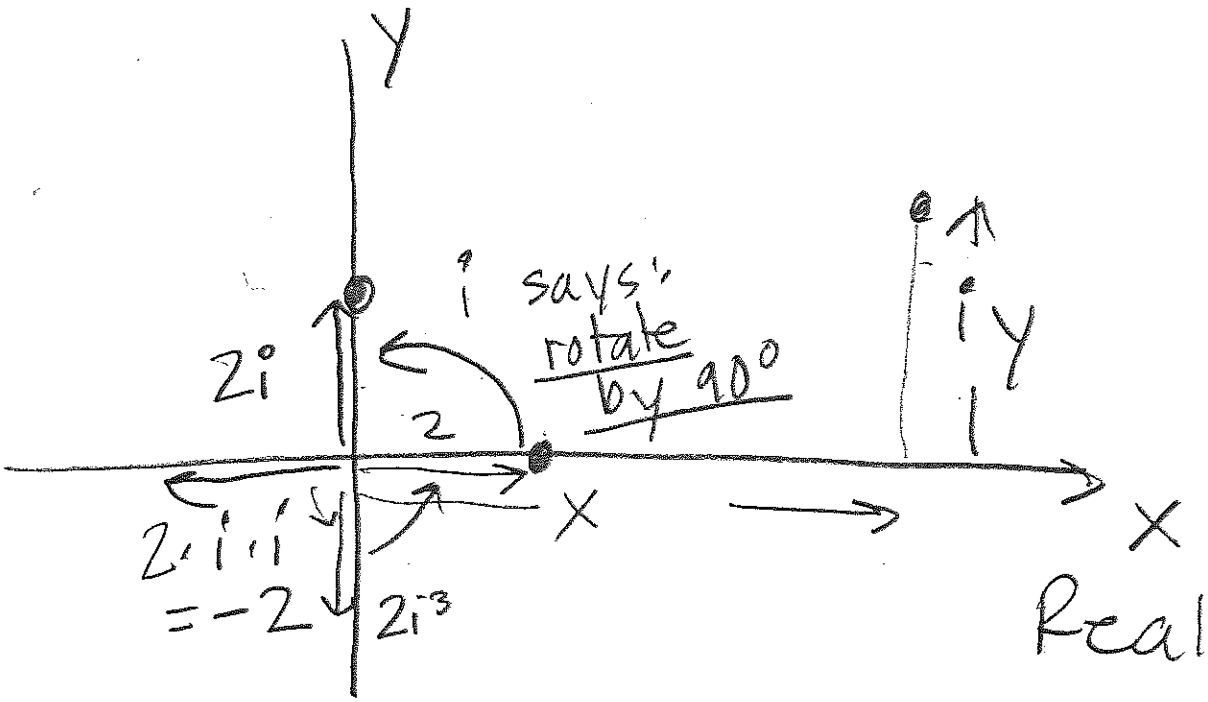
Complex Numbers

$\sqrt{-1} \equiv i$ "imaginary", so, $i^2 = -1$

Geometrically, i means: rotate by 90° ($\pi/2$) in the complex plane. Complex plane = Real x-axis

Imaginary y-axis.

$z = x + iy$ visualise as



So, say, $2 \cdot i$, read as:

2, rotated by 90°

$2 \cdot i \cdot i$ = 2 rotated twice by 90°

$= -2$

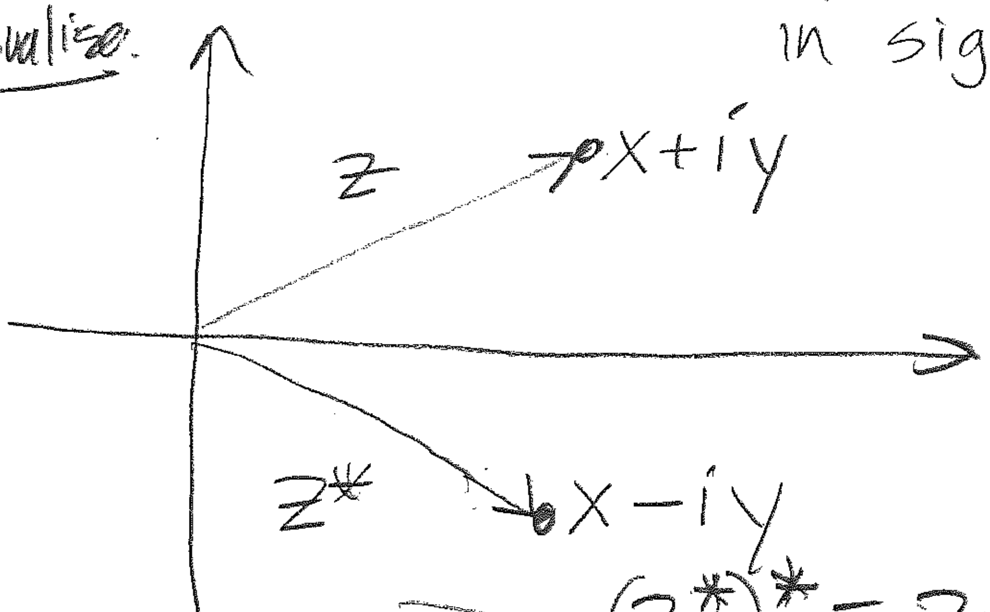
$2 \cdot i \cdot i \cdot i = -2i$

$2 \cdot i \cdot i \cdot i \cdot i = 2$

Complex Conjugation

$z^* \equiv x - iy$ (all imaginary parts reversed in sign)

visualise.



$(z^*)^* = z$

4 properties :

$z + z^* = 2x \equiv 2\,\mathrm{Re}(z)$

$z - z^* = 2iy = 2i\,\mathrm{Im}(y)$

$\sqrt{zz^*} = \sqrt{(x+iy)(x-iy)} = \sqrt{x^2+y^2}$

$= |z|$

$\sqrt{\dfrac{z}{z^*}} = ?$ How do you divide?

$\dfrac{3+4i}{2+i}$??? Multiply top + bottom by z^*

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$$\frac{3+4i}{2+i} = \frac{(3+4i)(2-i)}{2^2+1^2} = \frac{6-3i+8i+4}{5}$$

$$= \frac{10+5i}{5} = 2+i$$

$$(3+4i)(2+i)$$

$$\frac{3+4i}{2+i} = \quad = 6+3i+8i-4$$

$$= 2+11i$$

(A) $\frac{3}{2} + 4 - \frac{11}{2}$

(B) $2+11i$

(C) $2+i$

(D) 5

$$\frac{z}{z^*} = \frac{z \cdot z}{z^* \cdot z} = \frac{z^2}{|z|^2}$$

magnitude is ... one!

"A PURE ~~PHASE~~"