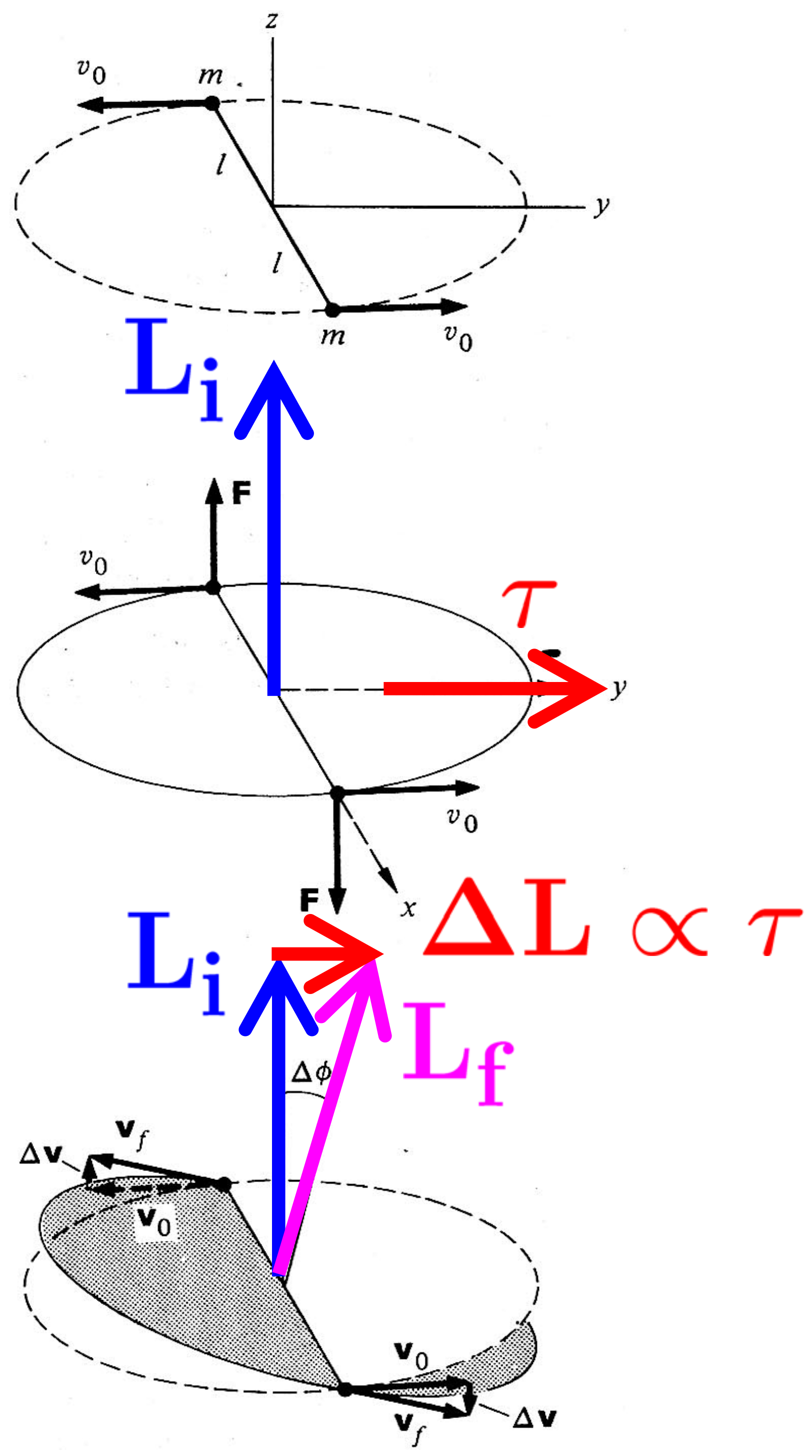
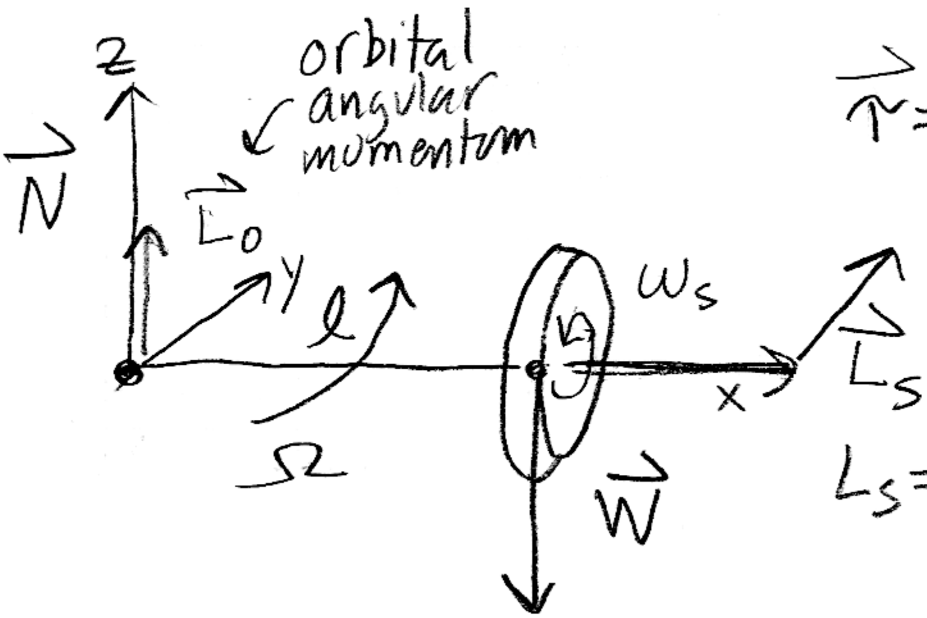
K&K Example 7.8

Qualitative
description of why
spinning
objects rotate in an
unexpected
manner



Gyroscope

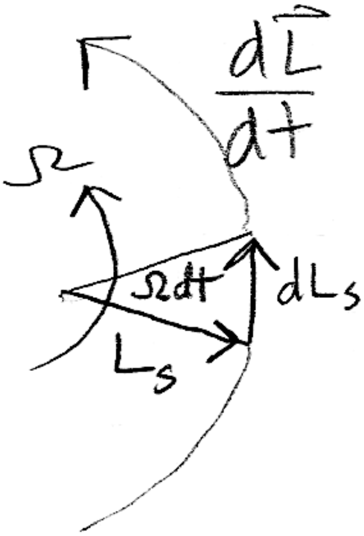


$$\vec{\tau} = \ell W \hat{j} = \frac{d\vec{L}}{dt}$$

("spin")

$$L_s = I_0 \omega_s$$

$$\vec{L} = \vec{L}_0 + \vec{L}_s$$



$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_s}{dt} = \ell W \hat{j}$$

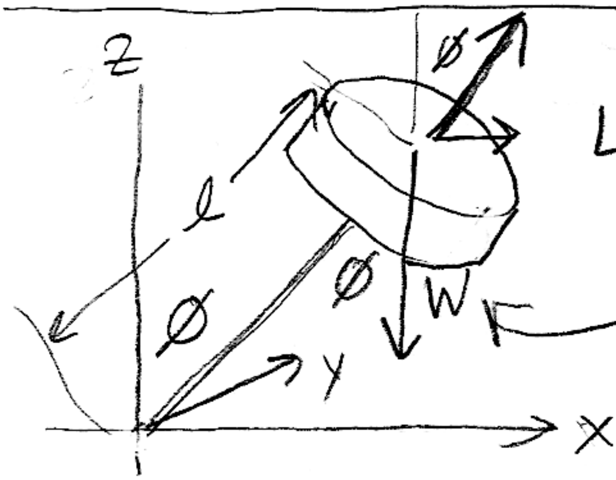
$$\left| \frac{dL_s}{L_s} \right| = \frac{\ell W \, dt}{I_0 \omega_s} = \Omega dt$$

$$\Omega = \frac{\ell W}{I_0 \omega_s} = \frac{\ell M g}{M k^2 \omega_s}$$

$$\boxed{\Omega = \frac{g \ell}{k^2 \omega_s}}$$

radius of gyration causing spin

Ω independent of "tip" angle!



$$L_{s\perp} = I_0 \omega_s \sin\phi$$

$$\vec{T} = \ell W \sin\phi \, \hat{J}$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_{s\perp}}{dt} = \ell W \sin\phi \, \hat{J}$$

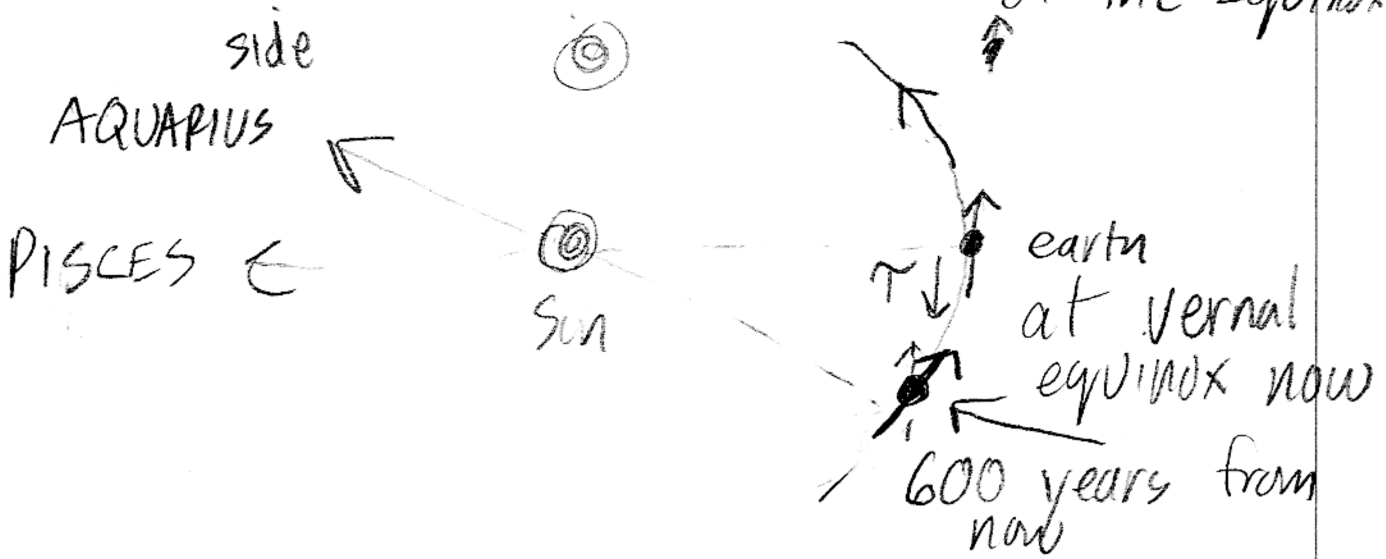
$$\Omega dt = \left| \frac{dL_{s\perp}}{L_{s\perp}} \right| = \frac{\ell W \sin\phi}{I_0 \omega_s \sin\phi} dt$$

$$\boxed{\Omega = \frac{\ell W}{I_0 \omega_s} = \frac{g \ell}{k^2 \omega_s}}$$

Precession of the Equinoxes

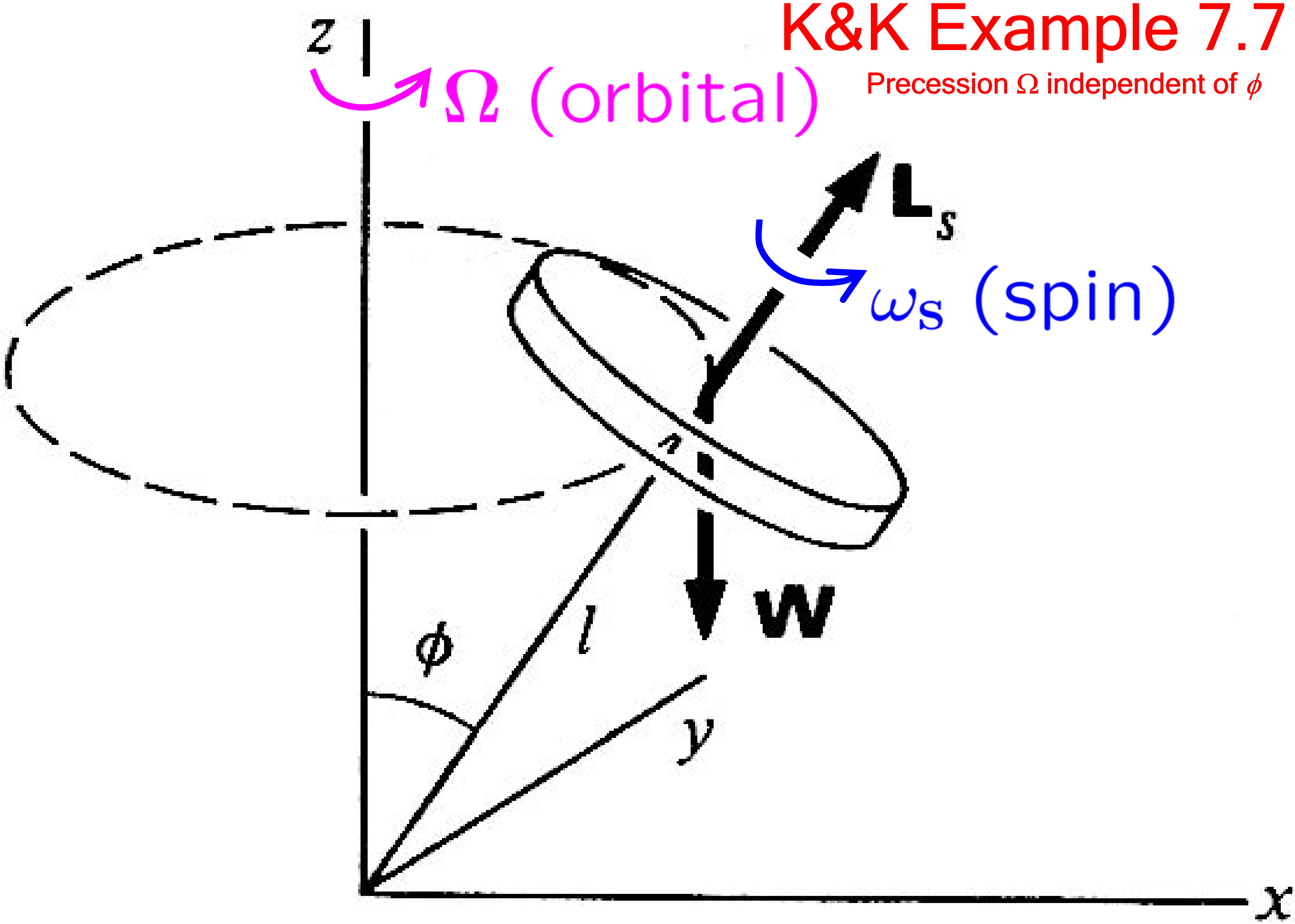
Period $\simeq$ 26,000 years

("Precession of the Equinoxes")

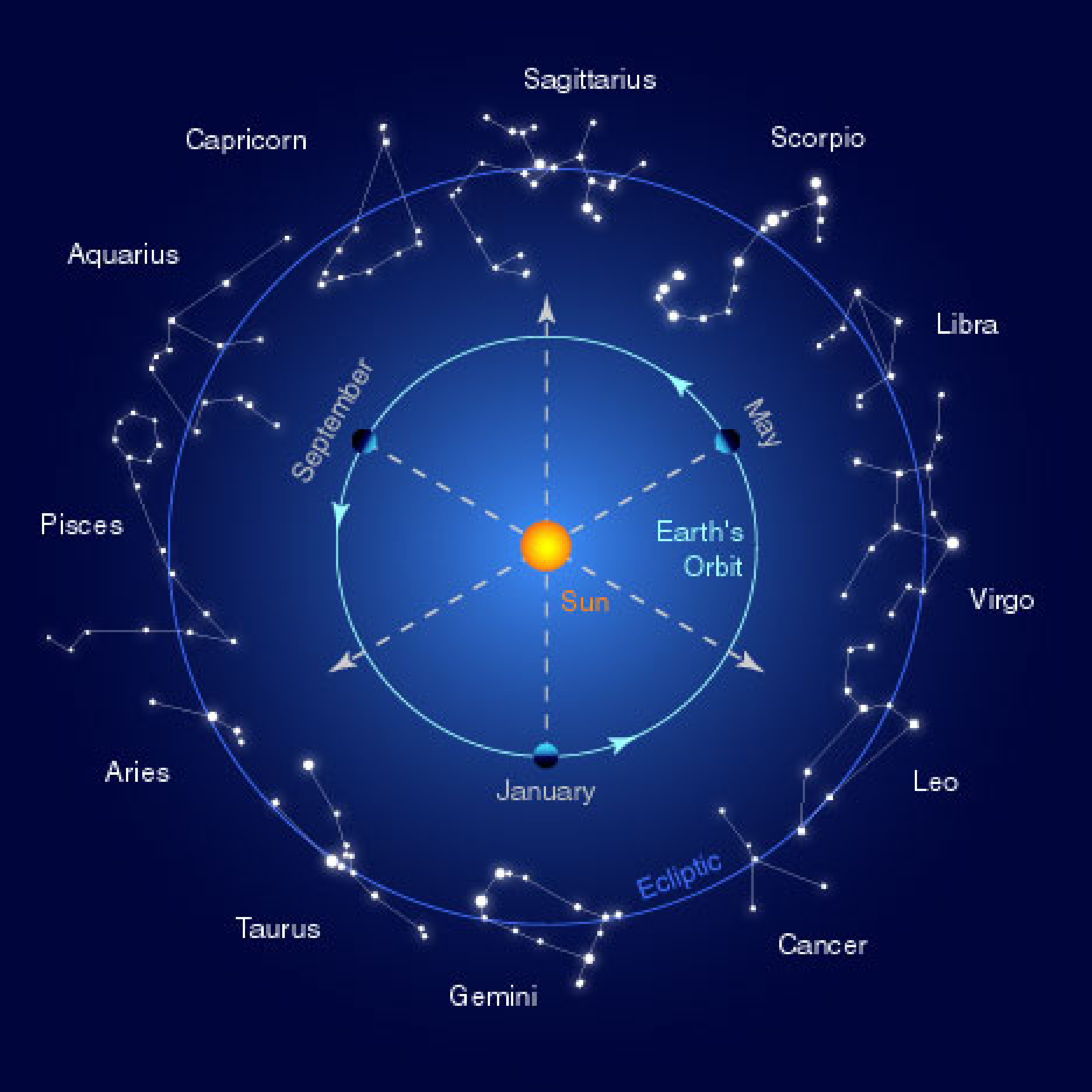


K&K Example 7.7

Precession Ω independent of ϕ

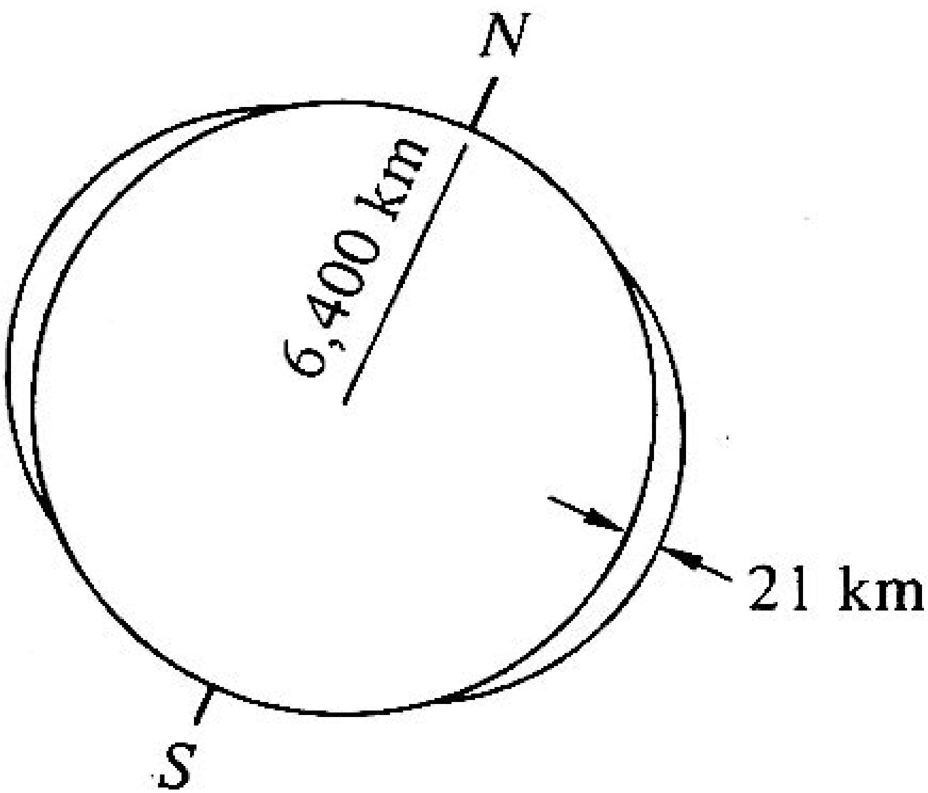


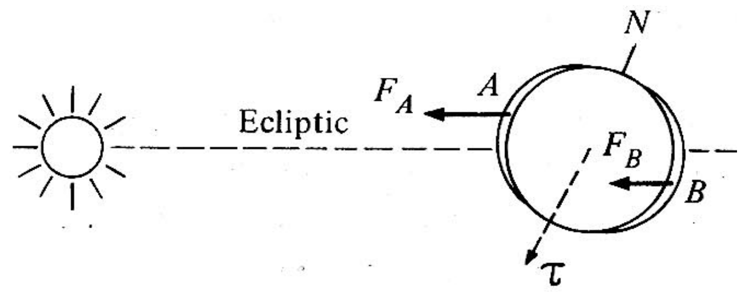
Constellation		Tropical date	Sidereal Date Cyril Fagan	IAU Definition Walter Berg
♈	Aries	March 21 - April 19	April 15 - May 15	April 19 - May 13 May 17 - May 18
♉	Taurus	April 20 - May 21	May 16 - June 15	May 14 - May 16 May 19 - June 19
♊	Gemini	May 22 - June 22	June 16 - July 15	June 20 - July 20
♋	Cancer	June 23 - July 22	July 16 - August 15	July 21 - August 9
♌	Leo	July 23 - August 22	August 16 - September 15	August 10 - September 15
♍	Virgo	August 23 - September 23	September 16 - October 15	September 16 - October 30
♎	Libra	September 24 - October 23	October 16 - November 15	October 31 - November 22
♏	Scorpius	October 24 - November 22	November 16 - December 15	November 23 - November 28
⚕/⛎	Ophiuchus	N/A		November 29 - December 17
♐	Sagittarius	November 23 - December 21	December 16 - January 14	December 18 - January 17
♑	Capricorn	December 22 - January 20	January 15 - February 14	January 18 - February 15
♒	Aquarius	January 21 - February 19	February 15 - March 14	February 16 - March 11
♓	Pisces	February 20 - March 20	March 15 - April 14	March 12 - April 18

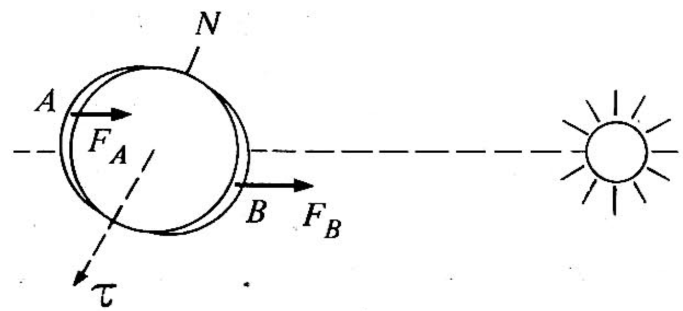

Sagittarius
Capricorn
Scorpio
Aquarius
Libra
September
May
Earth's
Orbit
Sun
Pisces
Virgo
Aries
Leo
January
Ecliptic
Taurus
Cancer
Gemini

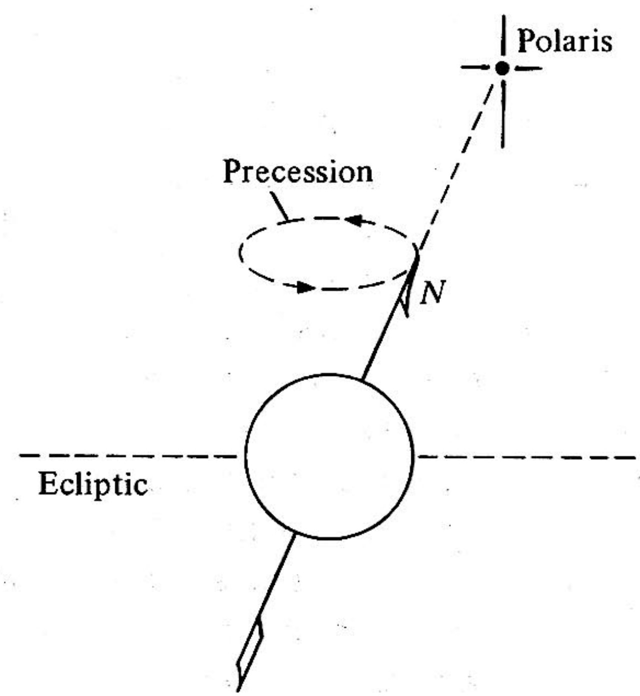
K&K Example 7.9

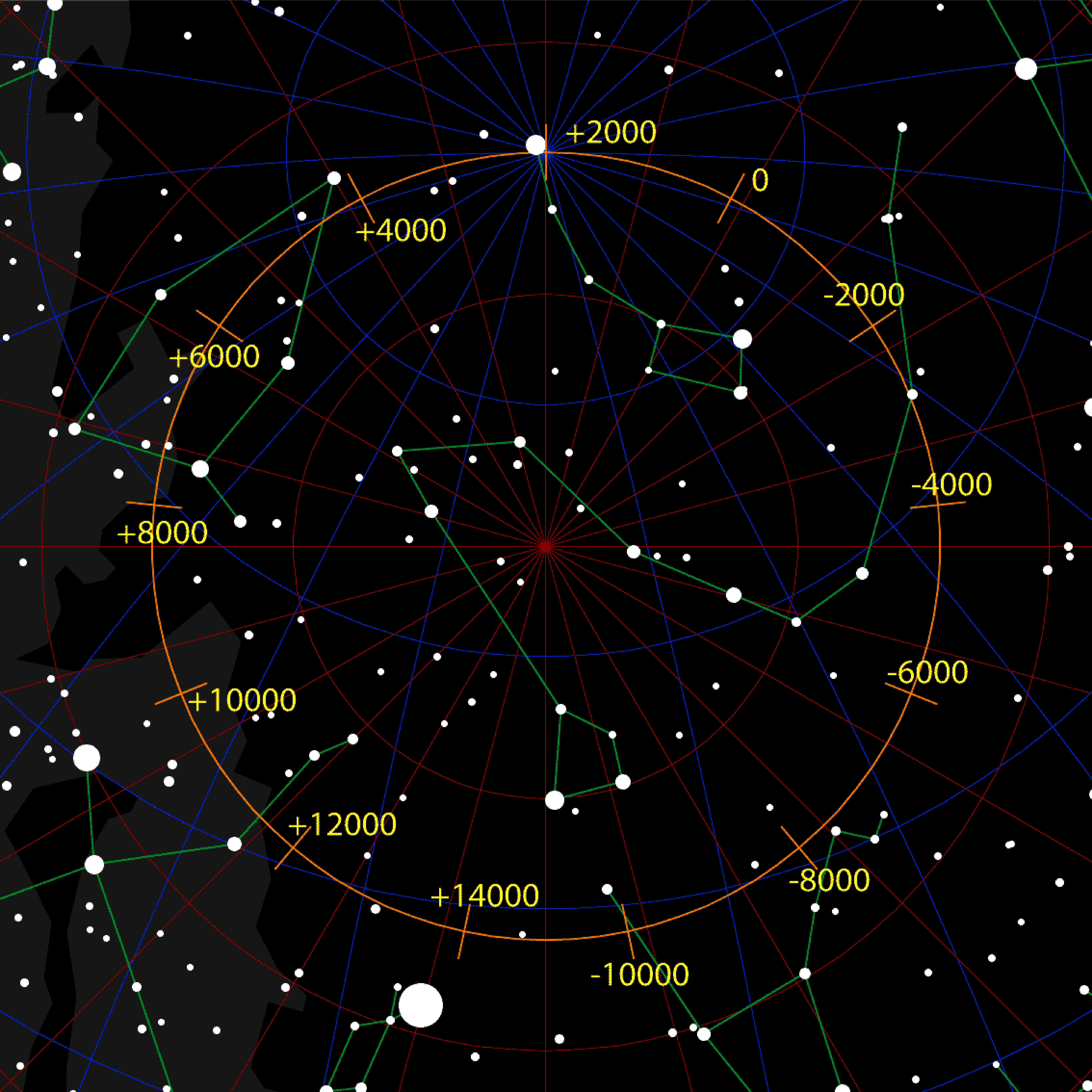
Precession of the equinoxes - Earth Precesses too











+2000
0
+4000
-2000
+6000
-4000
+8000
-6000
+10000
+12000
-8000
+14000
-10000

Statics

Rigid Bodies when

$$\sum \vec{F} = 0$$

$$\sum \vec{\tau} = 0$$

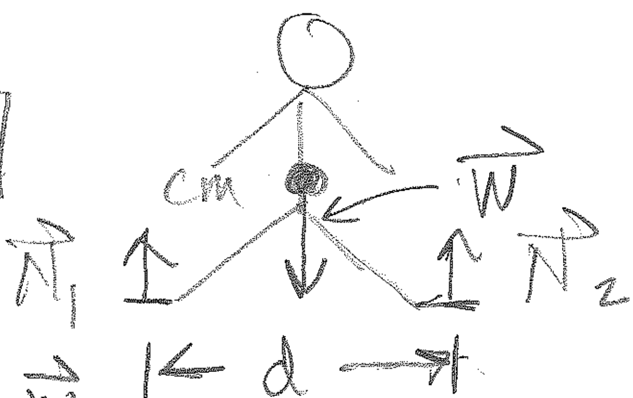
surprisingly interesting!

→ weight acts on center of mass, use that for torque.

Easy:

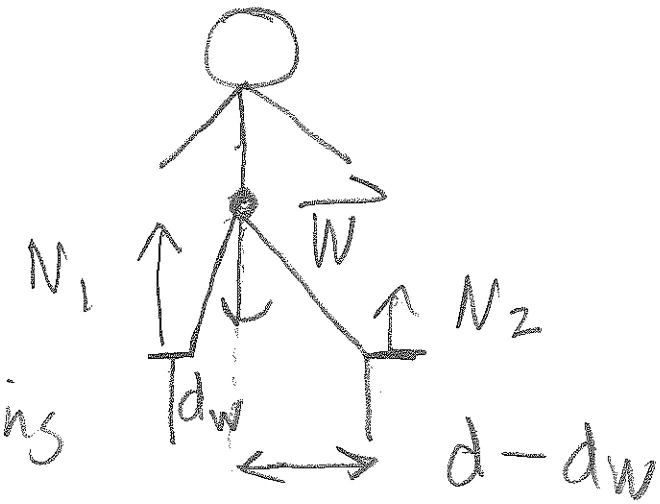
symmetric

↓



$$\vec{N}_1 = \vec{N}_2 = -\frac{\vec{W}}{2}$$

WHEN centered.



$\underline{\sum \vec{F}}$

$\underline{x}$: nothing

$\underline{y}$:

$$N_1 + N_2 - W = 0$$

$\sum \vec{\tau}$

$$N_2(d-d_w) - N_1 d_w = 0$$

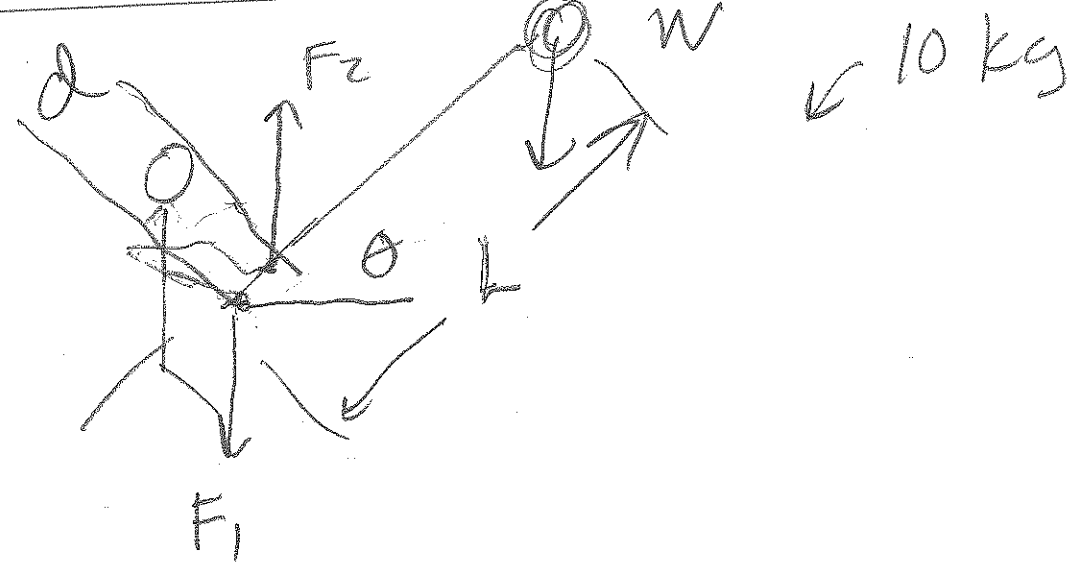
$$N_2 = \frac{d_w}{d-d_w} N_1$$

$$N_1 + \frac{d_w}{d-d_w} N_1 = W$$

$$\left(\frac{d - d_w + d_w}{d-d_w}\right) N_1 = W$$

$$N_1 = \left(\frac{d-d_w}{d}\right) W$$

$$\boxed{N_1 = \left(1 - \frac{d_w}{d}\right) W}$$



(1) $W + F_2 - F_1 = 0$

(2) person end of stick - pivot

$F_1 d\cos\theta - WL\cos\theta = 0$

$F_1 = \left(\frac{L}{d}\right) W$ big!

$F_2 = F_1 - W$

$F_2 = \left(\frac{L}{d}\right) W - W = \left(\frac{L}{d} - 1\right) W$

F_2 as pivot point

$F_2 d\cos\theta - W(L-d)\cos\theta = 0$

$F_2 = \left(\frac{L}{d} - 1\right) W$

$$N_2 = \frac{d_w}{d - d_w}, \quad N_1 = \frac{d_w}{d - d_w} \cdot \frac{d - d_w}{d} W$$

$$\boxed{N_2 = \frac{d_w}{d} W}$$