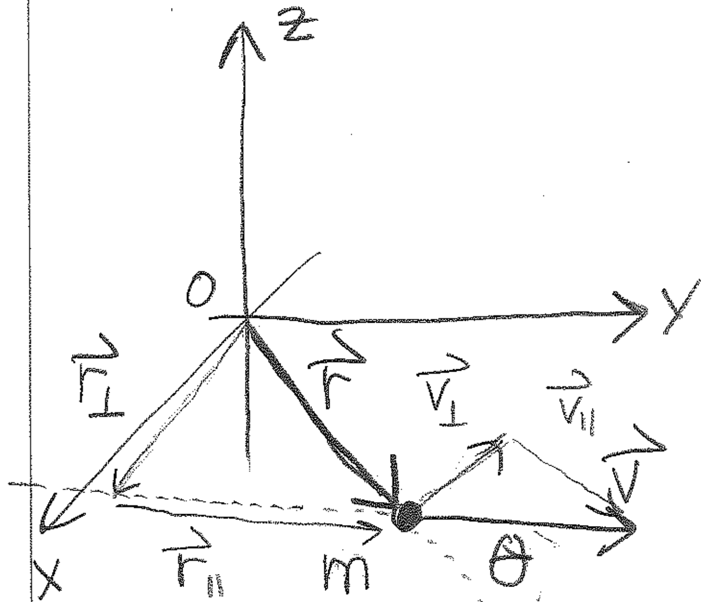
Angular Momentum

PHY4 Chapter 13, 6.2-6.3 K+K

Single particle. (3-d, hard!)



$\vec{r}$ + $\vec{v}$ in plane

$$\vec{L} = \vec{r} \times \underbrace{(m\vec{v})}_{\vec{p}}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

direction: z or $\hat{k}$

$$L = (rp\sin\theta)\hat{k}$$

$$= r_\perp p \hat{k} = r m v_\perp \hat{k}$$

here, $\vec{L}$ ∥ to $\underset{\omega}{\rightarrow}$ (not always true)

what good...

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

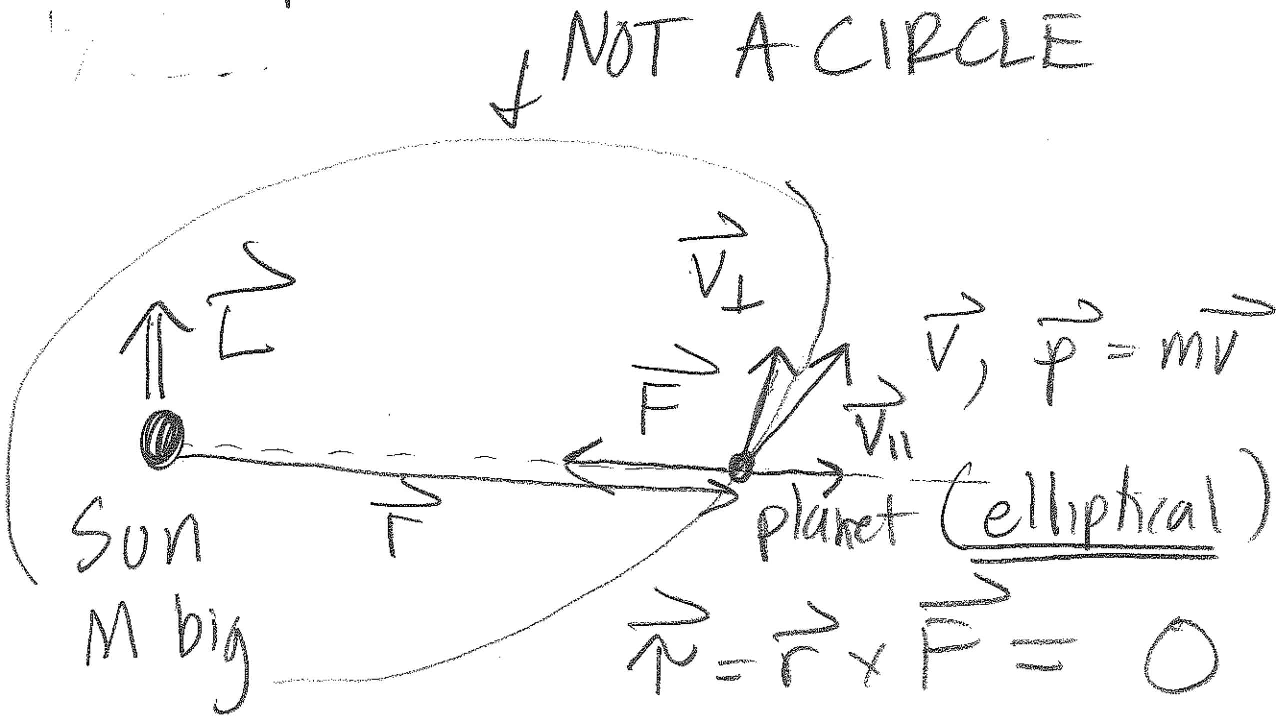
$$= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \underbrace{\vec{v} \times m\vec{v}}_{0} + \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

<u>Complication</u> ... might just change <u>direction</u> of $\vec{L}$ ... more later.

Most powerful ... "central" force

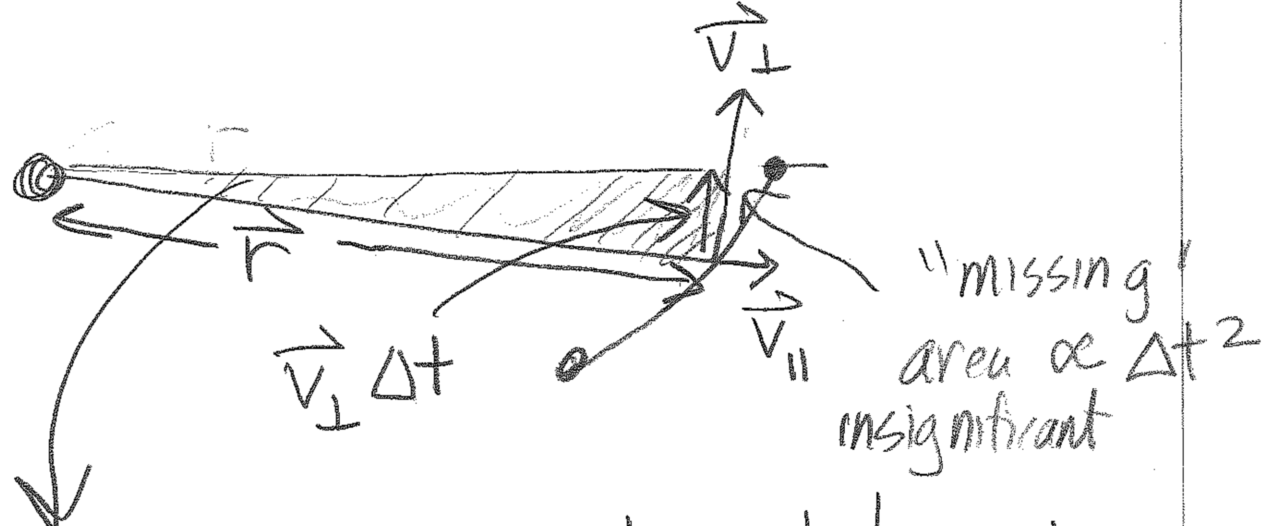


$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = 0$$

$\vec{L}$ = constant

= means, equal area swept out in equal (short) times.

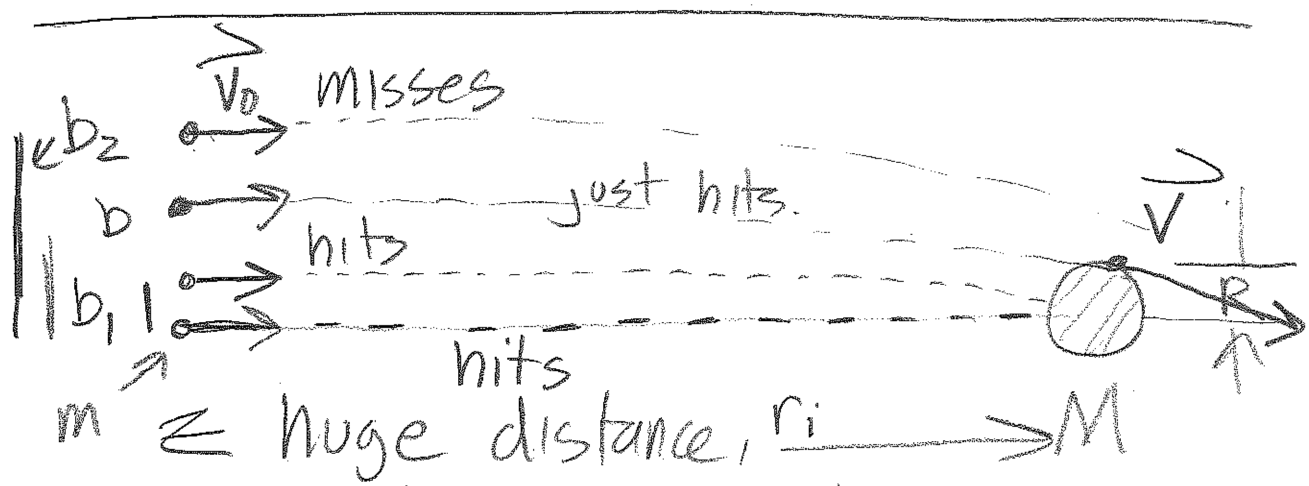


$$\Delta A = \text{area} = \frac{1}{2} r v_\perp \Delta t = \frac{1}{2} \frac{L}{m} \Delta t$$

$$\frac{\Delta A}{\Delta t} \Rightarrow \frac{dA}{dt} = \frac{1}{2} \frac{L}{m} = \text{constant}$$

"Kepler's 2ND Law of Planetary Motion" [Demo]

Capture Cross Section of a Planet



$b > R$

like planet has "bigger cross-section"

$\pi b^2 > \pi R^2$

$L = m v_0 b$ CONSERVED

$= m V R$ solve for b, need v!

Energy:

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_i} = \frac{1}{2} m v^2 - \frac{GMm}{R}$$

let $r_i = \infty$

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 + \frac{GMm}{R}$$

$$v^2 = v_0^2 + \frac{2GM}{R}$$

$$b = \frac{v}{v_0} R$$

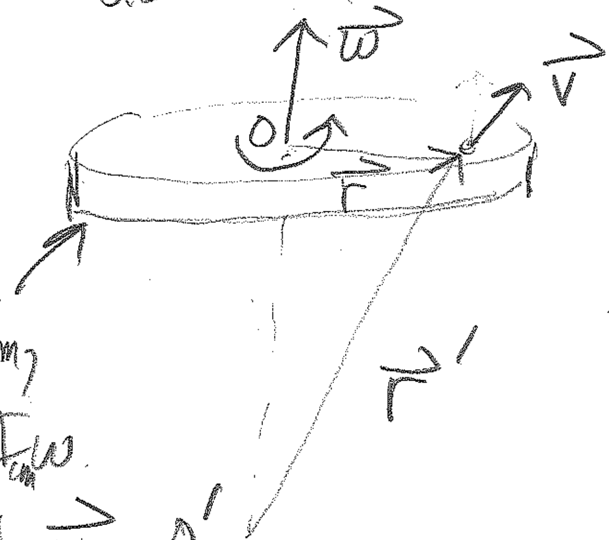
$$b = \left[1 + \frac{2GM}{R v_0^2}\right]^{1/2} R$$

note: $v_{esc}^2 = \frac{2GM}{R}$

$$b = \left[1 + \left(\frac{v_{esc}}{v_0}\right)^2\right]^{1/2} R, \quad A = \pi b^2 = \left[1 + \left(\frac{v_{esc}}{v_0}\right)^2\right] \pi R^2$$

Is $\vec{\omega} \parallel \vec{L}$?

<u>Yes when</u> a symmetric object rotates about its center.



$\vec{v} = \vec{\omega} \times \vec{r}$

<u>still</u>

$\vec{v} = \vec{\omega} \times \vec{r}\,'$

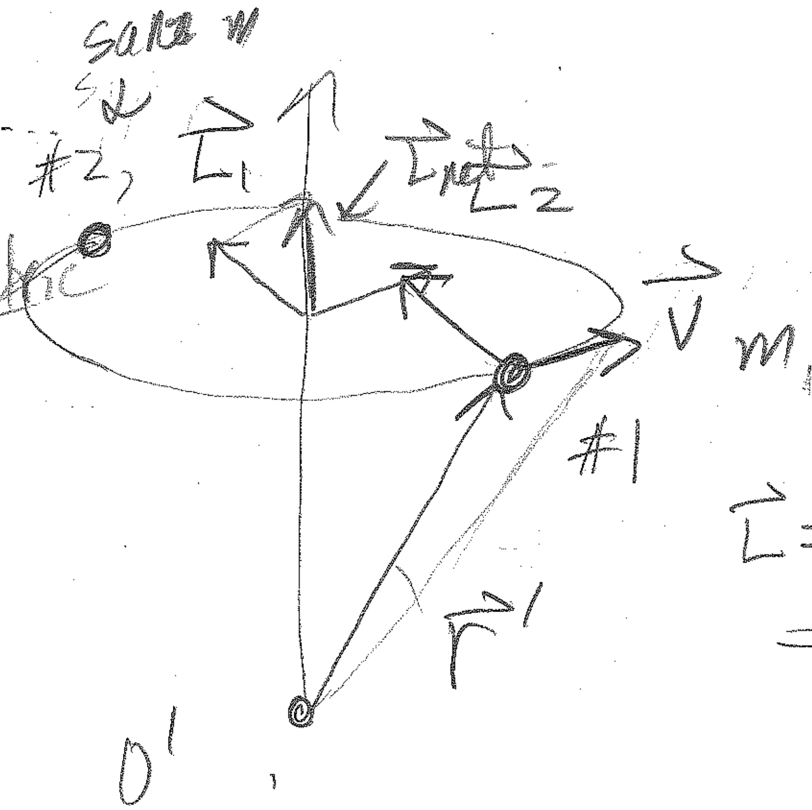
I_{cm},

$L = I_{cm}\omega$

$\vec{L} \parallel$ to $\vec{\omega}$

No...

Not symmetric



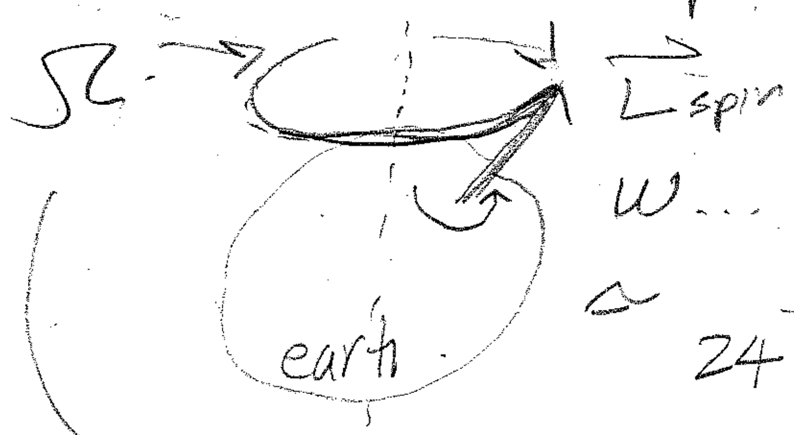
$\vec{L} = \vec{r}\,' \times \vec{p}$

$= \vec{r}\,' \times (m\vec{v})$

Earth

Earth Precession

Takes $\simeq$ 26,000 years for a complete round trip



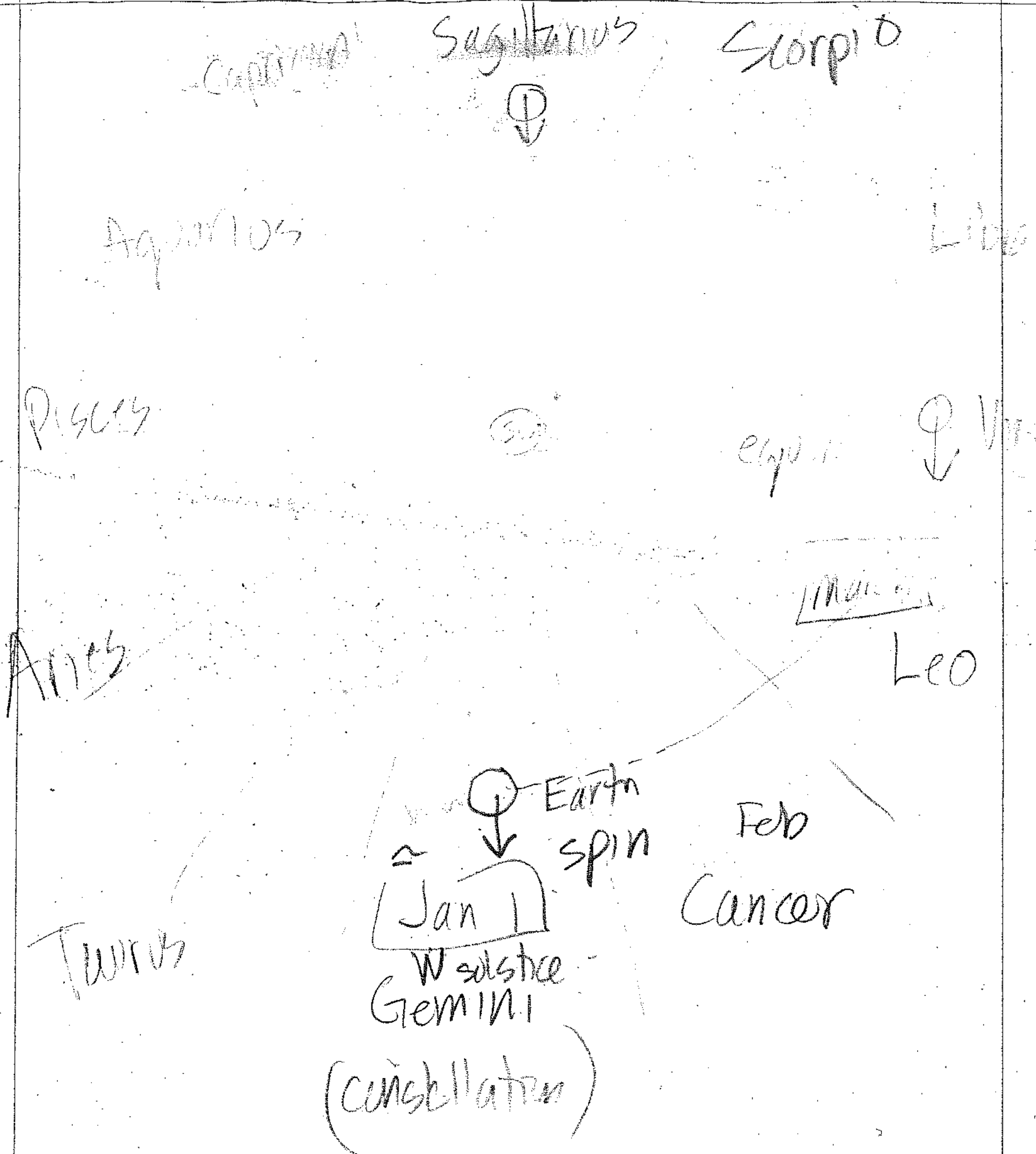
ω ... about 1/day $\simeq \frac{2\pi}{24 \cdot 60 \cdot 60}$ rad/sec

$$\Omega \simeq \frac{2\pi}{26{,}000 \text{ years}}$$

Oor year ("tropical") is

$\simeq$ winter solstice to winter solstice } growing seasons

$\neq$ 2π in azimuth about sun, due to precession!



Sagittarius
Scorpio
Aquarius
Pisces
Aries
Leo
Earth
spin
Feb
Jan 1
W solstice
Cancer
Taurus
Gemini
(constellation)