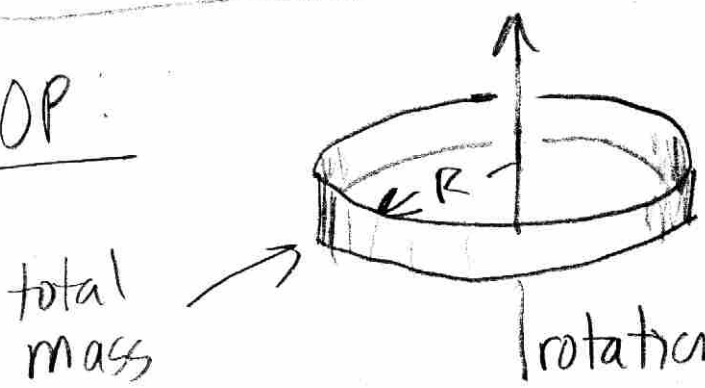
Rotational Inertia of Continous Mass Distributions

HOOP:



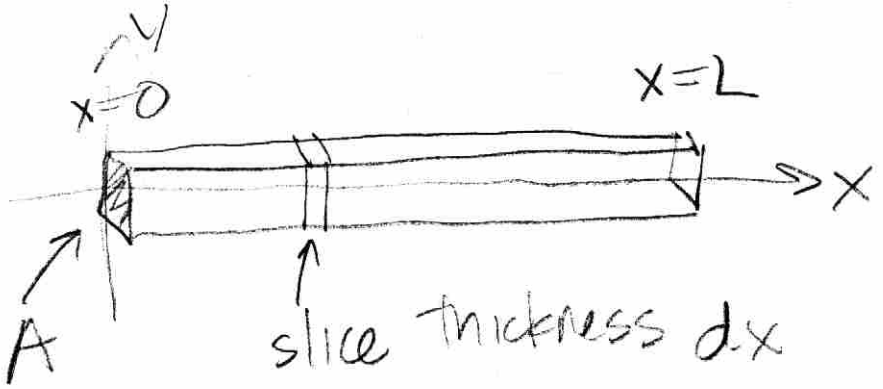
$I_{cm} = MR^2$ (as big as it gets)

Q: I if rotation axis goes through hoop edge? $I = I_{cm} + MR^2$
$= 2MR^2$

STICK:

Skip: 0 → Mass M
1 → center of Mass x_{cm}
2 → I_{cm}

M: assume constant mass density ρ



volume: $dV = A\,dx$

mass $dm = \rho\,dV = \rho A\,dx$

$$\underline{M} = \int dm = \rho A \int_0^L dx = \underline{\rho A L} \quad \rho \Leftrightarrow M$$

$$X_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\cancel{\rho A}\int_0^L x\,dx}{\cancel{\rho A} L} = \frac{\frac{1}{2}x^2\Big|_0^L}{L}$$

$$X_{cm} = \frac{1}{2}L$$

$$I_{cm} = \int_0^L (x - x_{cm})^2\,dm$$

$$= \rho A \int_0^L (x^2 - 2x x_{cm} + x_{cm}^2)\,dm$$

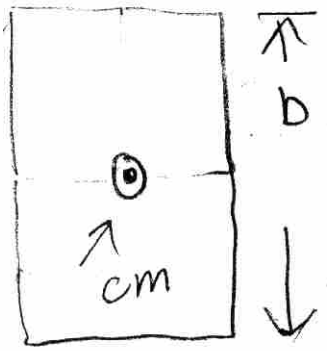
$$= \rho A \left[\frac{1}{3}x^3 - x_{cm}x^2 + x_{cm}^2 x \right]_0^L$$

$$= \rho A \left[\frac{1}{3}L^3 - \frac{L}{2}L^2 + \frac{L^2}{4}L \right]$$

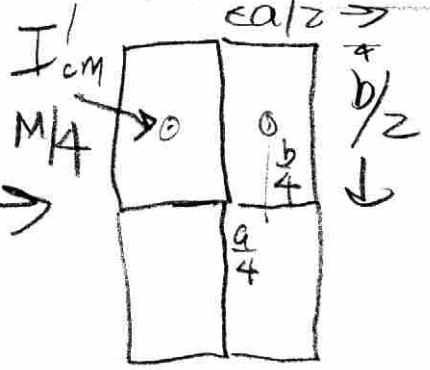
$$= \underbrace{\rho A L}_{M} \cdot \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{4} \right] L^2$$

$$\frac{4}{12} - \frac{6}{12} + \frac{3}{12} = \frac{1}{12}$$

$$I_{cm} = \frac{1}{12} M L^2$$



Naughty Trick →



$$I_{cm} = 16 I'_{cm} = 4\left[I'_{cm} + \frac{M}{4} \cdot \left(\left(\frac{a}{4}\right)^2 + \left(\frac{b}{4}\right)^2 \right) \right]$$

$16 = 4 \text{ (mass)} \cdot 2^2 \text{ (size)}$

$$4 I'_{cm} = I'_{cm} + \frac{M}{64}(a^2 + b^2)$$

$$3 I'_{cm} = \frac{M}{64}(a^2 + b^2)$$

$$I'_{cm} = \frac{M}{3 \cdot 64}(a^2+b^2)$$

$$I_{cm} = 16 I'_{cm} = \frac{16}{3 \cdot 64} M(a^2+b^2)$$

$$\boxed{I_{cm} = \frac{1}{12} M(a^2+b^2)}$$

RHK4 : Integral, p. 250