$$I_2 = m_1(-y_2)^2 + m_3\left((-y_2)^2 + (x_3)^2\right)$$
$$= 2\cdot 3^2 + 1\cdot 5^2$$
$$\boxed{I_2 = 18 + 25 = 43 \text{ kg}\cdot\text{m}^2}$$

$$I_3 = m_1(-x_3)^2 + m_2\left((-x_3)^2 + (y_2)^2\right)$$
$$= 2\cdot 4^2 + 3\cdot 5^2$$
$$\boxed{I_3 = 32 + 75 = 107 \text{ kg-m}^2}$$

When is I smallest? When rotation axis goes through the center of mass

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\overbrace{m_1\cdot 0 + m_2\cdot 0}^{0} + 1\cdot 4}{2+3+1}$$

m_1 m_2 count here

$$\boxed{x_{cm} = \frac{4}{6} = \frac{2}{3} \text{ m}}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{2\cdot 0 + 3\cdot 3 + 1\cdot 0}{2+3+1} = \frac{9}{6} \text{ m}$$

$$\boxed{y_{cm} = \frac{3}{2} \text{ m}}$$

$$|\vec{r}_1'| = \sqrt{\left(-\tfrac{2}{3}\right)^2 + \left(-\tfrac{3}{2}\right)^2} = 1.64\ m$$

$$|\vec{r}_2'| = \sqrt{\left(-\tfrac{2}{3}\right)^2 + \left(\tfrac{3}{2}\right)^2} = 1.64\ m$$

$$|\vec{r}_3'| = \sqrt{\left(3\tfrac{1}{3}\right)^2 + \left(-\tfrac{3}{2}\right)^2} = 3.66\ m$$

$$I_{cm} = m_1|\vec{r}_1'|^2 + m_2|\vec{r}_2'|^2 + m_3|\vec{r}_3{}^2|$$

$$= 2 \cdot 1.64^2 + 3 \cdot 1.64^2 + 1 \cdot 366^2$$

$$I_{cm} = 26.8\ kg-m^2$$

<u>Claim</u>: $I_1 = I_{cm} + (m_1+m_2+m_3)|\vec{r}_1|^2$

$$= 26.8 + (2+3+1)\cdot 1.64^2$$

$$= 43\ kg\cdot m^2 \checkmark$$

$$I_2 = I_{cm} + (m_1+m_2+m_3)|\vec{r}_2'|^2$$

$$= 43\ kg\ m^2 \checkmark$$

$$I_3 = I_{cm} + (m_1+m_2+m_3)|\vec{r}_3'|^2$$

$$= 107\ kg\cdot m^2 \checkmark$$

Known as the parallel axis theorem.

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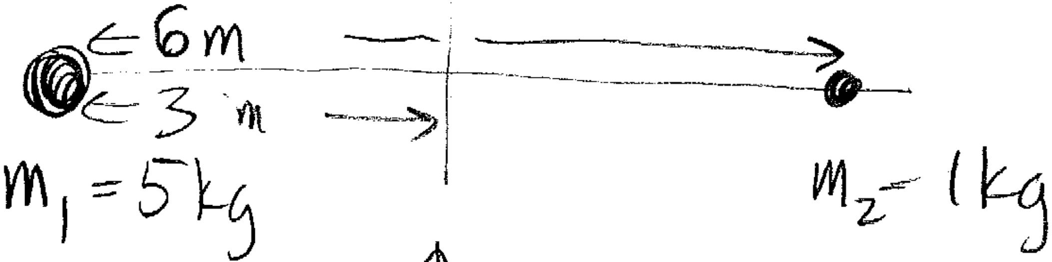


rotation axis

$I =$

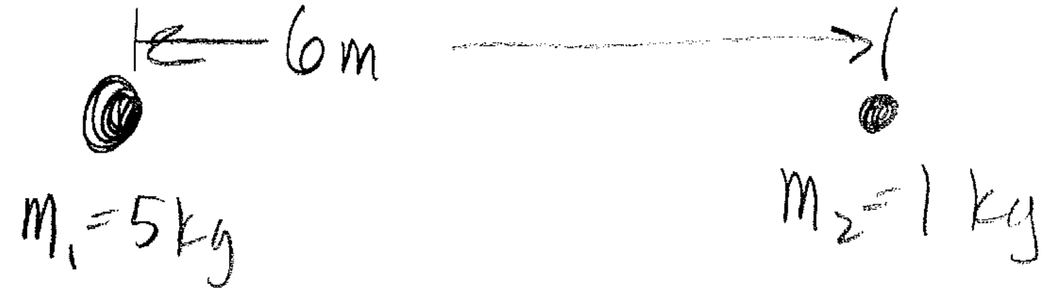
(A) 36 $kg \cdot m^2$

(B) 25 $kg \cdot m^2$

(C) 180 $kg \cdot m^2$

(D) 30 $kg \cdot m^2$

(E) 54 $kg \cdot m^2$



$I_{cm} =$

(A) 36 $kg \cdot m^2$

(B) 25 $kg \cdot m^2$

(C) 180 $kg \cdot m^2$

(D) 30 $kg \cdot m^2$

(E) 54 $kg \cdot m^2$

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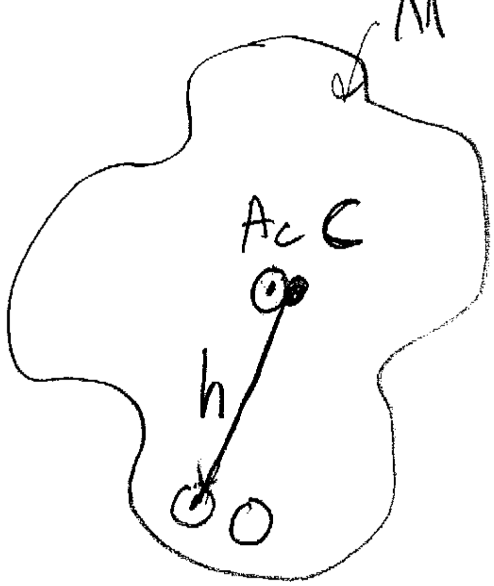
E/D

Parallel Axis Theorem



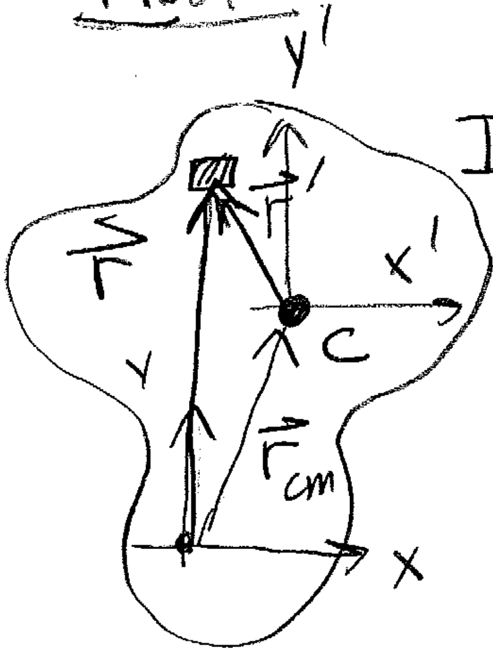
A_c : I_{cm}

rotate about O

$$I = I_{cm} + Mh^2$$

Proof

$$\vec{r}_{cm} + \vec{r}' = \vec{r}$$



$$I = \sum m_i |\vec{r}_i|^2$$

$$= \sum m_i |\vec{r}_{cm} + \vec{r}_i'|^2$$

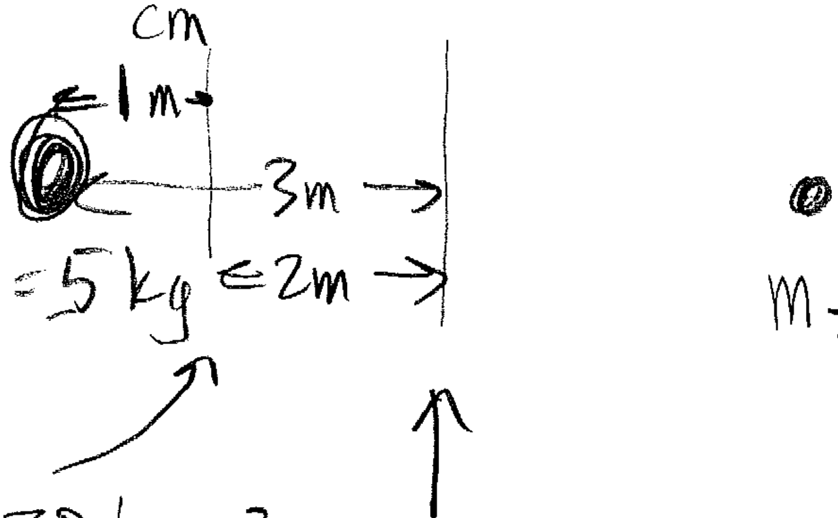
$$= \sum m_i (r_{cm}^2 + 2\vec{r}_{cm} \cdot \vec{r}_i' + r_i'^2)$$

$$\sum m_i \vec{r}_i' = 0$$

$$I = r_{cm}^2 \sum m_i + \sum m_i r_i'^2$$

$$I = h^2 \cdot M + I_{cm}$$

Previous Example



$m_1 = 5\text{ kg}$ $\qquad$ $m_2 = 1\text{ kg}$

$I_{cm} = 30\ \text{kg}\cdot\text{m}^2$ $\qquad$ $I = 54\ \text{kg}\cdot\text{m}^2$

$h = 2\text{ m}$

$$\underset{I}{54} \stackrel{?}{=} \underset{M}{6}\cdot\underset{h^2}{2^2} + \underset{I_{cm}}{30} \quad \text{kg}\cdot\text{m}^2$$

$$54 = 24 + 30 = 54\ \text{kg}\cdot\text{m}^2 \ \checkmark$$