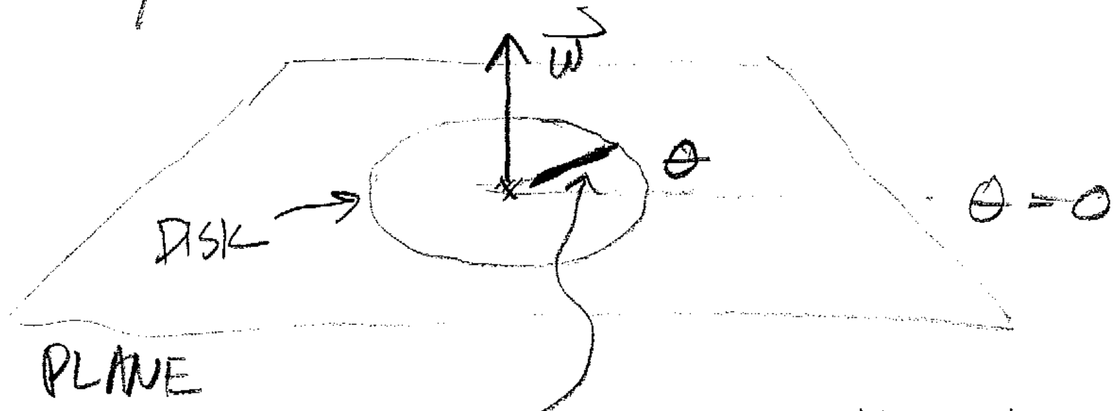
Angular Velocity Again

Easy case... rotation in one dimension



line ... θ like displacement

$\omega = \frac{d\theta}{dt}$ velocity.

$\omega > 0$ ... counterclockwise

$\omega < 0$ .. clockwise.

make ω into a vector... in this case, "RIGHT HAND RULE".. $|\vec{\omega}| = |\omega|$, direction $\perp$ to that plane.

Meaning: jump on disk, subtract $\vec{\omega}$

disk not rotating w/r to you!

Clock:

Second Hand: $\omega_s = -\frac{2\pi}{\cancel{60}\,30\ s}$

$= -1.05 \cdot 10^{-1}$ rad/s

Minute Hand $\omega_M = -\frac{2\pi}{60 \cdot 60} = -1.75 \cdot 10^{-3}$ rad/s

Rotations: DON'T COMMUTE

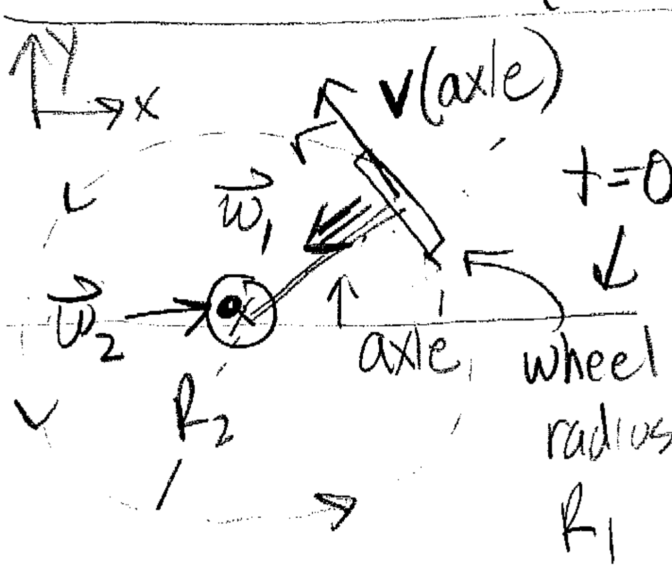
$\Rightarrow$ Described by matrizes.

As $\phi_1, \phi_2 \to 0$, they do commute.

so,
$$d\vec{\phi}_1 + d\vec{\phi}_2 = d\vec{\phi}_2 + d\vec{\phi}_1$$

$d\vec{\phi}$: size... $|d\vec{\phi}|$ small

direction: right hand rule.



$\vec{\omega}_1$: describes just wheel

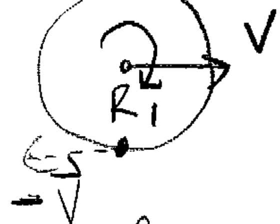
$\vec{\omega}_2$: describes wheel motion around center.

$$|\vec{\omega}_2| = \frac{2\pi}{T_2} = \frac{2\pi}{\left(\frac{2\pi R_2}{V}\right)} = \frac{V}{R_2}$$

$$V \cdot T_2 = 2\pi R_2$$

$$T_2 = \frac{2\pi R_2}{V}$$

$|\vec{\omega}_1|$?



seen from center

$$|\vec{\omega}_1| = \frac{V}{R_1}$$

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}$$

Take it a step further

$|\vec{w}(t)| = ?$

(A) $v\sqrt{\frac{1}{R_1^2}+\frac{1}{R_2^2}}$

(B) $v\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

(C) $v\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(D) $v\sqrt{\frac{1}{R_1^2}-\frac{1}{R_2^2}}$

(E) none of above.

$$\vec{\omega}(t) =$$

(A) $\frac{v}{R_1}\left(-\sin\left(\frac{v}{R_1}t\right)\hat{\imath} + \cos\left(\frac{v}{R_1}t\right)\hat{\jmath}\right) + \frac{v}{R_2}\hat{k}$

(B) $\frac{v}{R_1}\left(-\sin\left(\frac{v}{R_1}t\right)\hat{\imath} - \cos\left(\frac{v}{R_1}t\right)\hat{\jmath}\right) - \frac{v}{R_2}\hat{k}$

(C) $\frac{v}{R_1}\left(\cos\left(\frac{v}{R_1}t\right)\hat{\imath} + \sin\left(\frac{v}{R_1}t\right)\hat{\jmath}\right) + \frac{v}{R_2}\hat{k}$

(D) $\frac{v}{R_1}\left(-\cos\left(\frac{v}{R_1}t\right)\hat{\imath} - \sin\left(\frac{v}{R_1}t\right)\hat{\jmath}\right) + \frac{v}{R_2}\hat{k}$

(E) None of above.

$\theta \rightarrow$ not a vector

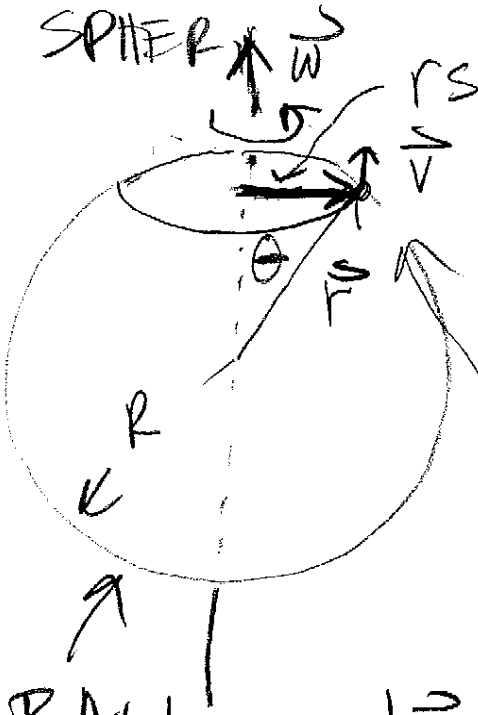
$\vec{\omega} \rightarrow$ a vector

$\vec{\alpha} = \frac{d\vec{\omega}}{dt} \rightarrow$ a vector.

what good is this?

$$\vec{V} = \vec{\omega} \times \vec{r}$$

point on rotating body — max be sum — points to spot on rotating body..



BALL

Note...

$VT = 2\pi r \sin\theta$

$v = \frac{2\pi}{T} r \sin\theta = \omega r \sin\theta$

$|\vec{\omega} \times \vec{r}| = \omega r \sin\theta \; (v)$

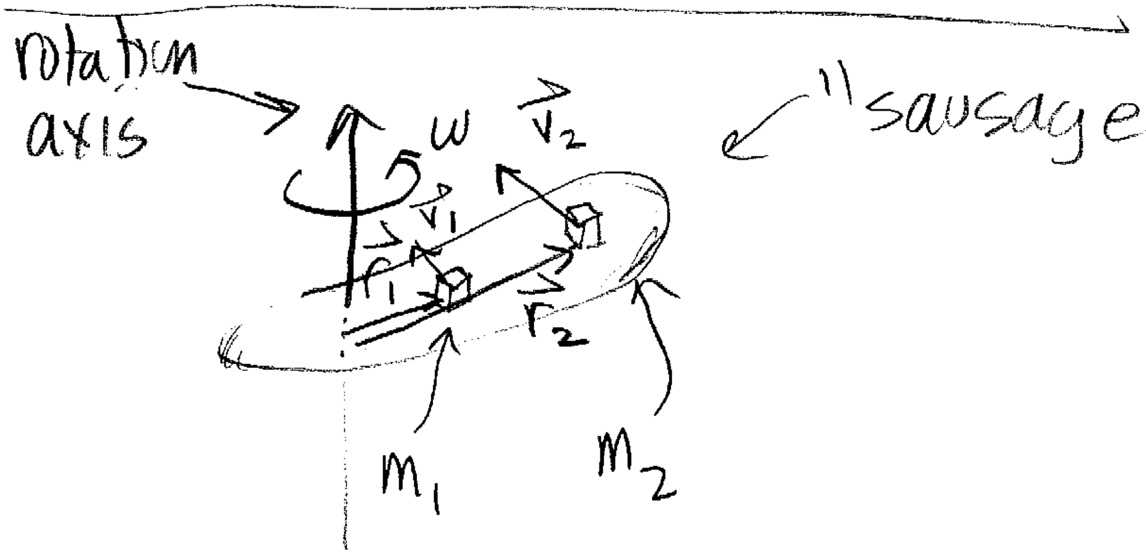
stare at direction!

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\vec{a} = \underbrace{\vec{\alpha} \times \vec{r}}_{\text{tangential}} + \underbrace{\vec{\omega} \times \vec{v}}_{\text{centripetal}}$$

$$= \underbrace{\vec{\alpha} \times \vec{r}}_{\simeq \perp \text{ to } \vec{r}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\simeq \parallel \text{ to } \vec{r}}$$

Kinetic Energy of Rotation and Rotational Inertia



$$K = \frac{1}{2} m_1 |\vec{v}_1|^2 + \frac{1}{2} m_2 |\vec{v}_2|^2 + \cdots$$

$$|\vec{\omega} \times \vec{r}_1|^2 \qquad |\vec{\omega} \times \vec{r}_2|$$

$$\omega^2 r_1^2 \qquad \omega^2 r_2^2$$

$$= \frac{1}{2}\left(\sum m_i r_i^2\right) \omega^2$$

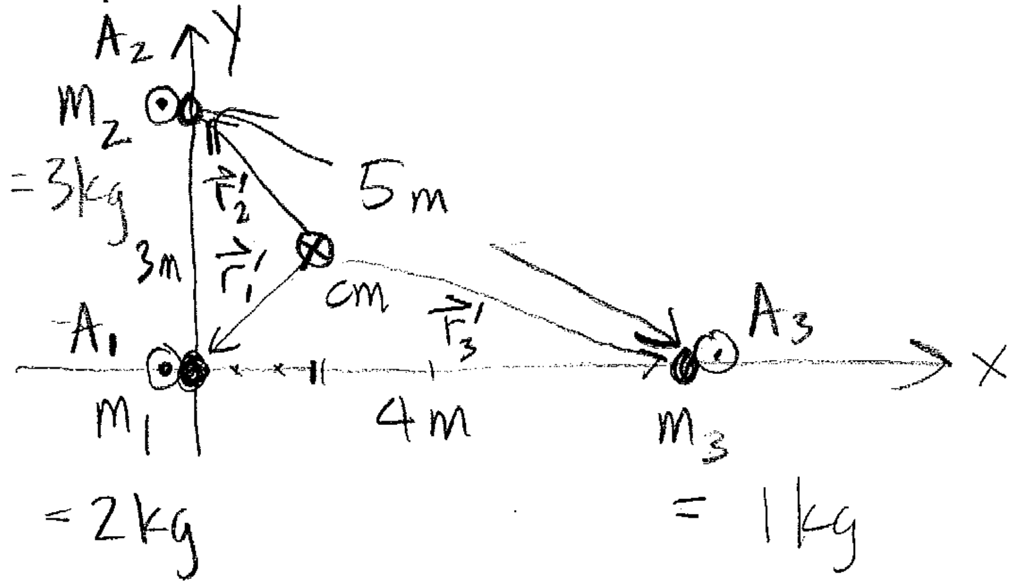
Rotational Inertia

$I = \sum m_i r_i^2 \Leftarrow$ depends on axis

then $K = \frac{1}{2} I \omega^2$

I is to m
as $\vec{\omega}$ is to $\vec{v}$

Example:



I about:

A_1: $I_1 = m_2 Y_2^2 + m_3 X_3^2$

$I_1 = \cdot 3 \cdot 3^2 + \cdot 1 \cdot 4^2$

$\boxed{I_1 = 27 + 16 = 43 \text{ kg} \cdot \text{m}^2}$