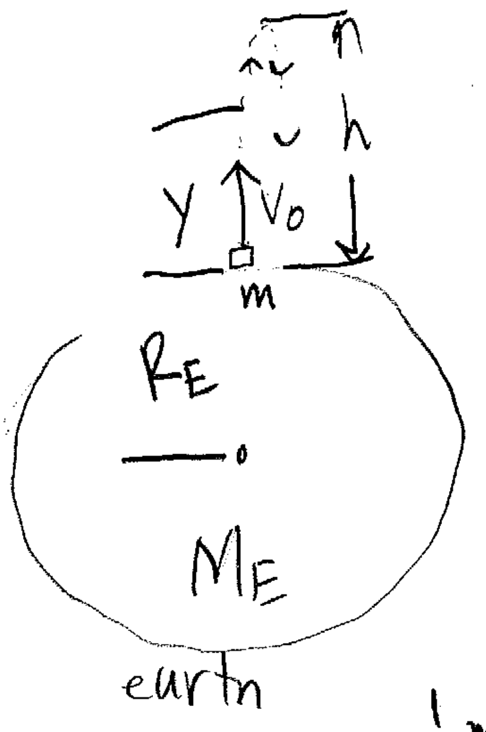


initial energy:

$$\frac{1}{2}mv_0^2 - \frac{GM\,m}{R_E}$$

energy at top, h

$$= 0 - \frac{GM\,m}{R_E + h}$$

$$\frac{1}{2}\cancel{m}v_0^2 - \frac{GM_E\cancel{m}}{R_E} = -\frac{GM_E\cancel{m}}{R_E + h}$$

from earlier... recall acceleration at Earth's surface is:

$$\cancel{m}g = \cancel{m}\frac{GM_E}{R_E^2}$$

$$\text{so } gR_E = \frac{GM_E}{R_E}$$

$$\frac{1}{2}v_0^2 - gR_E = -\frac{gR_E}{1 + h/R_E}$$

$$1 + \frac{h}{R_E} = \frac{gR_E}{gR_E - \frac{1}{2}v_0^2}$$

$$\frac{h}{R_E} = \frac{gR_E - gR_E + \frac{1}{2}v_0^2}{gR_E - \frac{1}{2}v_0^2}$$

$$h = \frac{\frac{v_0^2}{2g}}{1 - \frac{v_0^2}{2gR_E}}$$

← h_0, which would be height in uniform gravity (strength g)

$$h = \frac{h_0}{1 - \frac{h_0}{R_E}} > h_0$$

↑ because gravity weakens as you get higher

<u>Note</u>: when $1 - \frac{h_0}{R_E} = 0$

$$h \to \infty \; !$$

or $\frac{h_0}{R_E} = 1$

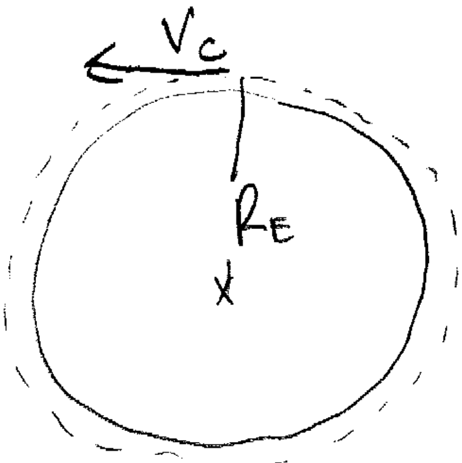
$$h_0 = \frac{v_0^2}{2g} = R_E$$

$$\boxed{V_0 \equiv V_{escape} = \sqrt{2gR_E} \simeq 1.1 \cdot 10^4 \ m/s}$$

Gun "Muzzle Velocity" $\sim 10^3$ m/s

<u>Moral</u>: DON'T FIRE UP... the bullet does not escape, but falls back to Earth, sometimes killing an innocent bystander!

Comparison... circular orbit at $r \simeq R_E$ ...



$$\frac{m v_c^2}{R_E} = m \frac{G M_E}{R_E^2} = mg$$

$$\boxed{v_c = \sqrt{g R_E} = \frac{1}{\sqrt{2}} v_{escape}}$$

In above, <u>neglected</u> Earth's rotation...

At equator, speed is....

$$= \frac{2\pi R_E}{T_{Day}} \simeq \frac{2\pi \cdot 6380 \cdot 10^3 \text{ m}}{24 \cdot 60 \cdot 60 \text{ s}}$$

$$\simeq 4{,}6 \cdot 10^2 \text{ m/s}$$

Helps enough that most launches of rockets go in the direction of earth's rotation to get extra "boost"!

Which brings up an interesting question ... suppose you launch ⊥ to vertical (neglecting Earth's rotation) ... is the escape velocity still $V_{escape} = \sqrt{2gR_E}$?

YES ... Why? The trajectory eventually looks vertical