In 2 or 3 dimensions ..

$$dW = \vec{F} \cdot d\vec{s} \text{ or } \vec{F} \cdot d\vec{r}$$

$d\vec{s} = d\vec{r}$ infinitesimal displacement

RHK4 KK

Think: circular orbit, gravity.

$\vec{F} \cdot d\vec{s} = 0$

Gravity does no work in this case.

Instaneous power

$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

true when $\vec{v}$, $\vec{F}$ not constant.

pp. 140-141 RHK4 p. 186 KK

Potential Energy U associated with conservative force field ...

1-d

$$U(x) = \underline{\text{constant}} + \int \underbrace{\left(-F(x)\right)}\, dx$$

F due to field

You would push with $-F$

arbitrary constant

$$\frac{dU}{dx} = -F(x) \quad \text{or} \quad F(x) = -\frac{dU}{dx}$$

Matters most is change in U

$$\Delta U = U(x) - U(x_0) = -\int_{x_0}^{x} F(x)\, dx$$

Gravity: $x \to y_y$ $\quad F(y) = -mg$

$$\Delta U = -\int_{y_0}^{y} (-mg)\, dy = mg\underbrace{(y - y_0)}_{h = y - y_0}$$

$$\Delta U = mgh$$

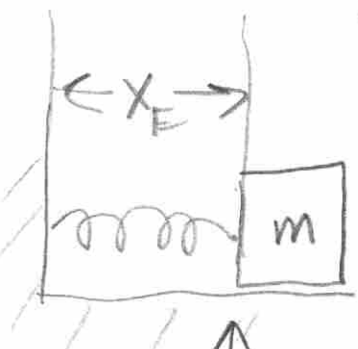
Work energy theorem ...

$$U(x_1) + K_1 = U(x_2) + K_2$$

Gravity $mgy_1 + \frac{1}{2}mv_1^2 = mgy_2 + \frac{1}{2}mv_2^2$

Simple Harmonic Oscillator ...

$$F = -k(x' - x_E)$$



↑ equilibrium length

just make
x = 0 the equilibrium length

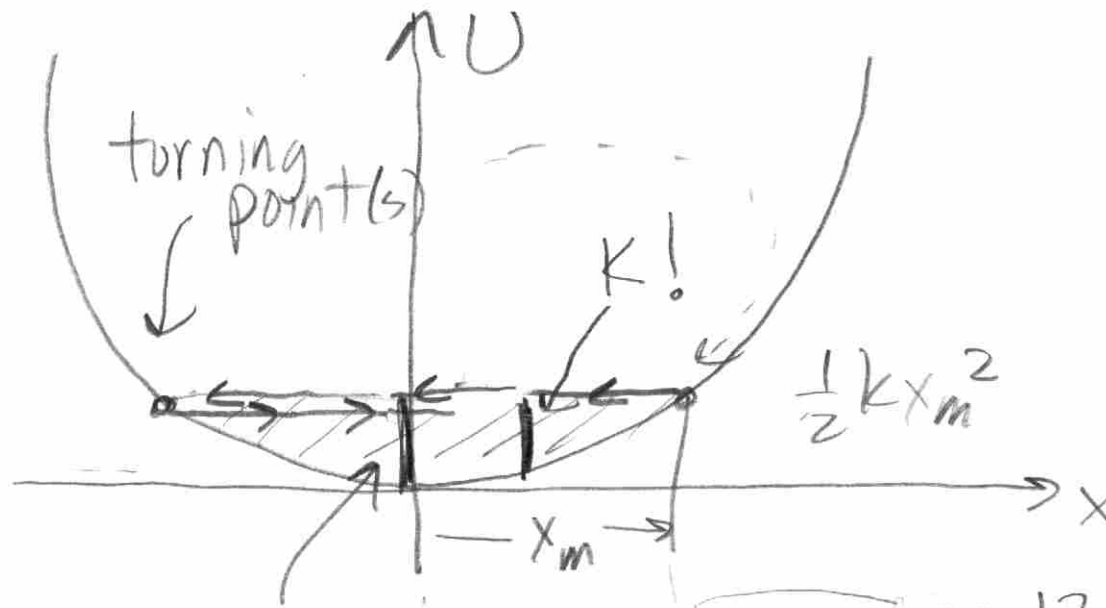
$$F = -kx$$

$$U(x) = \text{constant} + \int(--kx)\,dx$$

$$= \text{constant} + \frac{1}{2}kx^2$$

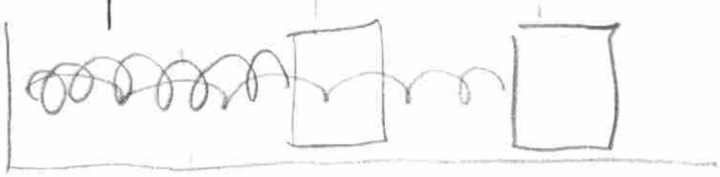
make a plot of this ...

make constant = 0



$PP\ 156-158$ RHK4 | $pp.\ 176-182$ K+K

Imagine... pulling spring to maximum x_m displacement from equilibrium



$$U(x_m) = \frac{1}{2}kx_m^2 = E = U(x) + K$$

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + K$$

$$K = \frac{1}{2}k(x_m^2 - x^2)$$

"Motion" -- oscillation between turning points ...

$$v_m \rightarrow \quad \frac{1}{2} m v_m^2 = \frac{1}{2} k x_m^2$$

$$v_m = \pm \sqrt{\frac{k}{m}}\, x_m$$

$$v_m = \pm \omega_0 x_m$$

$$T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{k}}.$$

More generally ... when $U(x)$ has a minimum ...

$$(N) \equiv x \longrightarrow (N)$$

N_2 nitrogen $\simeq 78\%$ of E

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

x small ... $\rightarrow \dfrac{a}{x^{12}}$

x large $\rightarrow -\dfrac{b}{x^6}$